International Tables for Crystallography (2006). Vol. A1, Maximal subgroups of space group 155, pp. 296–297.

$R32$ No. 155 $R32$ D_3^7

RHOMBOHEDRAL AXES

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; (2); (4)

General position

Multiplicity, Wyckoff letter, Site symmetry			Coordinates		
6	f	1	(1) x,y,z	(2) z,x,y	(3) y,z,x
			(4) $\bar{z},\bar{y},\bar{x}$	(5) $\bar{y},\bar{x},\bar{z}$	(6) $\bar{x},\bar{z},\bar{y}$

I Maximal *translationengleiche* subgroups

[2] $R31$ (146, $R3$)	1; 2; 3		
[3] $R12$ (5, $C121$)	1; 4	$-\mathbf{a}-\mathbf{c},-\mathbf{a}+\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	
[3] $R12$ (5, $C121$)	1; 5	$-\mathbf{a}-\mathbf{b},\mathbf{a}-\mathbf{b},\mathbf{a}+\mathbf{b}+\mathbf{c}$	
[3] $R12$ (5, $C121$)	1; 6	$-\mathbf{b}-\mathbf{c},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	

II Maximal *klassengleiche* subgroups

- **Loss of centring translations** none

- **Enlarged unit cell**

[2] $\mathbf{a}'=\mathbf{a}+\mathbf{c},\ \mathbf{b}'=\mathbf{a}+\mathbf{b},\ \mathbf{c}'=\mathbf{b}+\mathbf{c}$

$R32$ (155)	$\langle 2;\ 4\rangle$	$\mathbf{a}+\mathbf{c},\mathbf{a}+\mathbf{b},\mathbf{b}+\mathbf{c}$	
$R32$ (155)	$\langle 2;\ 4+(1,1,1)\rangle$	$\mathbf{a}+\mathbf{c},\mathbf{a}+\mathbf{b},\mathbf{b}+\mathbf{c}$	$1/2,1/2,1/2$

[3] $\mathbf{a}'=\mathbf{a}-\mathbf{b},\ \mathbf{b}'=\mathbf{b}-\mathbf{c},\ \mathbf{c}'=\mathbf{a}+\mathbf{b}+\mathbf{c}$

$P3_221$ (154)	$\langle 4;\ 2+(2,0,0)\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	$2/3,0,-2/3$
$P3_221$ (154)	$\langle (2;\ 4)+(1,0,1)\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	$2/3,0,1/3$
$P3_221$ (154)	$\langle 2+(1,1,0);\ 4+(1,2,1)\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	$2/3,1,1/3$
$P3_121$ (152)	$\langle 4;\ 2+(1,0,0)\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	$1/3,0,-1/3$
$P3_121$ (152)	$\langle 2+(1,-1,1);\ 4+(2,0,2)\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	$4/3,0,2/3$
$P3_121$ (152)	$\langle 2+(1,1,-1);\ 4+(0,2,0)\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	$1/3,1,-1/3$
$P321$ (150)	$\langle 2;\ 4\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	
$P321$ (150)	$\langle 2+(1,-1,0);\ 4+(1,0,1)\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	$1,0,0$
$P321$ (150)	$\langle 2+(1,0,-1);\ 4+(1,2,1)\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	$1,1,0$

[4] $\mathbf{a}'=\mathbf{a}-\mathbf{b}+\mathbf{c},\ \mathbf{b}'=\mathbf{a}+\mathbf{b}-\mathbf{c},\ \mathbf{c}'=-\mathbf{a}+\mathbf{b}+\mathbf{c}$

$R32$ (155)	$\langle 2;\ 4\rangle$	$\mathbf{a}-\mathbf{b}+\mathbf{c},\mathbf{a}+\mathbf{b}-\mathbf{c},-\mathbf{a}+\mathbf{b}+\mathbf{c}$	
$R32$ (155)	$\langle (2;\ 4)+(1,-2,1)\rangle$	$\mathbf{a}-\mathbf{b}+\mathbf{c},\mathbf{a}+\mathbf{b}-\mathbf{c},-\mathbf{a}+\mathbf{b}+\mathbf{c}$	$1,-1,0$
$R32$ (155)	$\langle 2+(1,1,-2);\ 4+(-1,2,-1)\rangle$	$\mathbf{a}-\mathbf{b}+\mathbf{c},\mathbf{a}+\mathbf{b}-\mathbf{c},-\mathbf{a}+\mathbf{b}+\mathbf{c}$	$0,1,-1$
$R32$ (155)	$\langle 4;\ 2+(2,-1,-1)\rangle$	$\mathbf{a}-\mathbf{b}+\mathbf{c},\mathbf{a}+\mathbf{b}-\mathbf{c},-\mathbf{a}+\mathbf{b}+\mathbf{c}$	$1,0,-1$

- **Series of maximal isomorphic subgroups**

$[p]$ $\mathbf{a}'=\frac{1}{3}\big((p+1)\mathbf{a}+(p-2)\mathbf{b}+(p+1)\mathbf{c}\big),\ \mathbf{b}'=\frac{1}{3}\big((p+1)\mathbf{a}+(p+1)\mathbf{b}+(p-2)\mathbf{c}\big),\ \mathbf{c}'=\frac{1}{3}\big((p-2)\mathbf{a}+(p+1)\mathbf{b}+(p+1)\mathbf{c}\big)$

$R32$ (155) $\langle 2;\ 4+(2u,2u,2u)\rangle$ $\mathbf{a}'=\frac{1}{3}\big((p+1)\mathbf{a}\ldots$, see lattice relations u,u,u
$p>4;\ 0\le u<p$
p conjugate subgroups for prime $p\equiv 2 \pmod 3$

$[p]$ $\mathbf{a}'=\frac{1}{3}\big((p+2)\mathbf{a}+(p-1)\mathbf{b}+(p-1)\mathbf{c}\big),\ \mathbf{b}'=\frac{1}{3}\big((p-1)\mathbf{a}+(p+2)\mathbf{b}+(p-1)\mathbf{c}\big),\ \mathbf{c}'=\frac{1}{3}\big((p-1)\mathbf{a}+(p-1)\mathbf{b}+(p+2)\mathbf{c}\big)$

$R32$ (155) $\langle 2;\ 4+(2u,2u,2u)\rangle$ $\mathbf{a}'=\frac{1}{3}\big((p+2)\mathbf{a}\ldots$, see lattice relations u,u,u
$p>6;\ 0\le u<p$
p conjugate subgroups for prime $p\equiv 1 \pmod 3$

$[p^2]$ $\mathbf{a}'=\frac{1}{3}\big((p+1)\mathbf{a}+(1-2p)\mathbf{b}+(p+1)\mathbf{c}\big),\ \mathbf{b}'=\frac{1}{3}\big((p+1)\mathbf{a}+(p+1)\mathbf{b}+(1-2p)\mathbf{c}\big),\ \mathbf{c}'=\frac{1}{3}\big((1-2p)\mathbf{a}+(p+1)\mathbf{b}+(p+1)\mathbf{c}\big)$

$R32$ (155) $\langle 2+(u+v,-2u+v,u-2v);$ $\mathbf{a}'=\frac{1}{3}\big((p+1)\mathbf{a}\ldots$, see lattice relations
$4+(u-v,-2u+2v,u-v)\rangle$ $u,-u+v,-v$
$p>1;\ 0\le u<p;\ 0\le v<p$
p^2 conjugate subgroups for prime $p\equiv 2 \pmod 3$

$[p^2]$ $\mathbf{a}'=\frac{1}{3}\big((2p+1)\mathbf{a}+(1-p)\mathbf{b}+(1-p)\mathbf{c}\big),\ \mathbf{b}'=\frac{1}{3}\big((1-p)\mathbf{a}+(2p+1)\mathbf{b}+(1-p)\mathbf{c}\big),\ \mathbf{c}'=\frac{1}{3}\big((1-p)\mathbf{a}+(1-p)\mathbf{b}+(2p+1)\mathbf{c}\big)$

$R32$ (155) $\langle 2+(u+v,-2u+v,u-2v);$ $\mathbf{a}'=\frac{1}{3}\big((2p+1)\mathbf{a}\ldots$, see lattice relations
$4+(u-v,-2u+2v,u-v)\rangle$ $u,-u+v,-v$
$p>6;\ 0\le u<p;\ 0\le v<p$
p^2 conjugate subgroups for prime $p\equiv 1 \pmod 3$

I Minimal *translationengleiche* supergroups

[2] $R\bar{3}m$ (166); [2] $R\bar{3}c$ (167); [4] $P432$ (207); [4] $P4_232$ (208); [4] $F432$ (209); [4] $F4_132$ (210); [4] $I432$ (211); [4] $P4_332$ (212); [4] $P4_132$ (213); [4] $I4_132$ (214)

II Minimal non-isomorphic *klassengleiche* supergroups

- **Additional centring translations** none
- **Decreased unit cell**

[3] $\mathbf{a}' = \frac{1}{3}(2\mathbf{a}-\mathbf{b}-\mathbf{c}),\ \mathbf{b}' = \frac{1}{3}(-\mathbf{a}+2\mathbf{b}-\mathbf{c}),\ \mathbf{c}' = \frac{1}{3}(\mathbf{a}+\mathbf{b}+\mathbf{c})$ $P312$ (149)