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D_{3d}^{6} $R\bar{3}2/c$ No. 167 $R\bar{3}c$

RHOMBOHEDRAL AXES

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; (2); (4); (7)

General position

Multiplicity, Wyckoff letter, Site symmetry	Coordinates		
12 f 1	(1) x,y,z	(2) z,x,y	(3) y,z,x
	(4) $\bar{z}+\frac{1}{2},\bar{y}+\frac{1}{2},\bar{x}+\frac{1}{2}$	(5) $\bar{y}+\frac{1}{2},\bar{x}+\frac{1}{2},\bar{z}+\frac{1}{2}$	(6) $\bar{x}+\frac{1}{2},\bar{z}+\frac{1}{2},\bar{y}+\frac{1}{2}$
	(7) $\bar{x},\bar{y},\bar{z}$	(8) $\bar{z},\bar{x},\bar{y}$	(9) $\bar{y},\bar{z},\bar{x}$
	(10) $z+\frac{1}{2},y+\frac{1}{2},x+\frac{1}{2}$	(11) $y+\frac{1}{2},x+\frac{1}{2},z+\frac{1}{2}$	(12) $x+\frac{1}{2},z+\frac{1}{2},y+\frac{1}{2}$

I Maximal *translationengleiche* subgroups

[2] $R3c$ (161)	1; 2; 3; 10; 11; 12		
[2] $R32$ (155)	1; 2; 3; 4; 5; 6		1/4, 1/4, 1/4
[2] $R\bar{3}1$ (148, $R\bar{3}$)	1; 2; 3; 7; 8; 9		
{ [3] $R12/c$ (15, $C12/c1$)	1; 4; 7; 10	$-\mathbf{a}-\mathbf{c},-\mathbf{a}+\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	
{ [3] $R12/c$ (15, $C12/c1$)	1; 5; 7; 11	$-\mathbf{a}-\mathbf{b},\mathbf{a}-\mathbf{b},\mathbf{a}+\mathbf{b}+\mathbf{c}$	
{ [3] $R12/c$ (15, $C12/c1$)	1; 6; 7; 12	$-\mathbf{b}-\mathbf{c},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	

II Maximal *klassengleiche* subgroups

- **Loss of centring translations** none
- **Enlarged unit cell**

[3] $\mathbf{a}'=\mathbf{a}-\mathbf{b}$, $\mathbf{b}'=\mathbf{b}-\mathbf{c}$, $\mathbf{c}'=\mathbf{a}+\mathbf{b}+\mathbf{c}$

{ $P\bar{3}c1$ (165)	$\langle 2;\ 4;\ 7\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	
{ $P\bar{3}c1$ (165)	$\langle 2+(1,-1,0);\ 4+(1,0,1);\ 7+(2,0,0)\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	1, 0, 0
{ $P\bar{3}c1$ (165)	$\langle 2+(1,0,-1);\ 4+(1,2,1);\ 7+(2,2,0)\rangle$	$\mathbf{a}-\mathbf{b},\mathbf{b}-\mathbf{c},\mathbf{a}+\mathbf{b}+\mathbf{c}$	1, 1, 0

[4] $\mathbf{a}'=\mathbf{a}-\mathbf{b}+\mathbf{c}$, $\mathbf{b}'=\mathbf{a}+\mathbf{b}-\mathbf{c}$, $\mathbf{c}'=-\mathbf{a}+\mathbf{b}+\mathbf{c}$

{ $R\bar{3}c$ (167)	$\langle 2;\ 4;\ 7\rangle$	$\mathbf{a}-\mathbf{b}+\mathbf{c},\mathbf{a}+\mathbf{b}-\mathbf{c},-\mathbf{a}+\mathbf{b}+\mathbf{c}$	
{ $R\bar{3}c$ (167)	$\langle (2;\ 4)+(1,-2,1);\ 7+(2,-2,0)\rangle$	$\mathbf{a}-\mathbf{b}+\mathbf{c},\mathbf{a}+\mathbf{b}-\mathbf{c},-\mathbf{a}+\mathbf{b}+\mathbf{c}$	1, −1, 0
{ $R\bar{3}c$ (167)	$\langle 2+(1,1,-2);\ 4+(-1,2,-1);\ 7+(0,2,-2)\rangle$	$\mathbf{a}-\mathbf{b}+\mathbf{c},\mathbf{a}+\mathbf{b}-\mathbf{c},-\mathbf{a}+\mathbf{b}+\mathbf{c}$	0, 1, −1
{ $R\bar{3}c$ (167)	$\langle 4;\ 2+(2,-1,-1);\ 7+(2,0,-2)\rangle$	$\mathbf{a}-\mathbf{b}+\mathbf{c},\mathbf{a}+\mathbf{b}-\mathbf{c},-\mathbf{a}+\mathbf{b}+\mathbf{c}$	1, 0, −1

- **Series of maximal isomorphic subgroups**

[p] $\mathbf{a}'=\frac{1}{3}\big((p+1)\mathbf{a}+(p-2)\mathbf{b}+(p+1)\mathbf{c}\big)$, $\mathbf{b}'=\frac{1}{3}\big((p+1)\mathbf{a}+(p+1)\mathbf{b}+(p-2)\mathbf{c}\big)$, $\mathbf{c}'=\frac{1}{3}\big((p-2)\mathbf{a}+(p+1)\mathbf{b}+(p+1)\mathbf{c}\big)$

$R\bar{3}c$ (167) $\langle 2;\ 4+(\frac{p}{2}-\frac{1}{2}+2u,\frac{p}{2}-\frac{1}{2}+2u,\frac{p}{2}-\frac{1}{2}+2u);$ $7+(2u,2u,2u)\rangle$ $\mathbf{a}'=\frac{1}{3}\big((p+1)\mathbf{a}\ldots$, see lattice relations u,u,u
$p>4$; $0\le u<p$
p conjugate subgroups for prime $p\equiv 5 \pmod 6$

[p] $\mathbf{a}'=\frac{1}{3}\big((p+2)\mathbf{a}+(p-1)\mathbf{b}+(p-1)\mathbf{c}\big)$, $\mathbf{b}'=\frac{1}{3}\big((p-1)\mathbf{a}+(p+2)\mathbf{b}+(p-1)\mathbf{c}\big)$, $\mathbf{c}'=\frac{1}{3}\big((p-1)\mathbf{a}+(p-1)\mathbf{b}+(p+2)\mathbf{c}\big)$

$R\bar{3}c$ (167) $\langle 2;\ 4+(\frac{p}{2}-\frac{1}{2}+2u,\frac{p}{2}-\frac{1}{2}+2u,\frac{p}{2}-\frac{1}{2}+2u);$ $7+(2u,2u,2u)\rangle$ $\mathbf{a}'=\frac{1}{3}\big((p+2)\mathbf{a}\ldots$, see lattice relations u,u,u
$p>6$; $0\le u<p$
p conjugate subgroups for prime $p\equiv 1 \pmod 6$

[p^2] $\mathbf{a}'=\frac{1}{3}\big((p+1)\mathbf{a}+(1-2p)\mathbf{b}+(p+1)\mathbf{c}\big)$, $\mathbf{b}'=\frac{1}{3}\big((p+1)\mathbf{a}+(p+1)\mathbf{b}+(1-2p)\mathbf{c}\big)$, $\mathbf{c}'=\frac{1}{3}\big((1-2p)\mathbf{a}+(p+1)\mathbf{b}+(p+1)\mathbf{c}\big)$

$R\bar{3}c$ (167) $\langle 2+(u+v,-2u+v,u-2v);$ $4+(u-v,-2u+2v,u-v);$ $7+(2u,-2u+2v,-2v)\rangle$ $\mathbf{a}'=\frac{1}{3}\big((p+1)\mathbf{a}\ldots$, see lattice relations $u,-u+v,-v$
$p>1$; $0\le u<p$; $0\le v<p$
p^2 conjugate subgroups for prime $p\equiv 2 \pmod 3$

[p^2] $\mathbf{a}'=\frac{1}{3}\big((2p+1)\mathbf{a}+(1-p)\mathbf{b}+(1-p)\mathbf{c}\big)$, $\mathbf{b}'=\frac{1}{3}\big((1-p)\mathbf{a}+(2p+1)\mathbf{b}+(1-p)\mathbf{c}\big)$, $\mathbf{c}'=\frac{1}{3}\big((1-p)\mathbf{a}+(1-p)\mathbf{b}+(2p+1)\mathbf{c}\big)$

$R\bar{3}c$ (167) $\langle 2+(u+v,-2u+v,u-2v);$ $4+(u-v,-2u+2v,u-v);$ $7+(2u,-2u+2v,-2v)\rangle$ $\mathbf{a}'=\frac{1}{3}\big((2p+1)\mathbf{a}\ldots$, see lattice relations $u,-u+v,-v$
$p>6$; $p\equiv 1 \pmod 3$; $0\le u<p$; $0\le v<p$
p^2 conjugate subgroups for prime $p\equiv 1 \pmod 3$

I Minimal *translationengleiche* supergroups

[4] $Pn\bar{3}n$ (222); [4] $Pm\bar{3}n$ (223); [4] $Fm\bar{3}c$ (226); [4] $Fd\bar{3}c$ (228); [4] $Ia\bar{3}d$ (230)

II Minimal non-isomorphic *klassengleiche* supergroups

- **Additional centring translations** none

- **Decreased unit cell**

[3] $\mathbf{a}' = \frac{1}{3}(2\mathbf{a}-\mathbf{b}-\mathbf{c}),\ \mathbf{b}' = \frac{1}{3}(-\mathbf{a}+2\mathbf{b}-\mathbf{c}),\ \mathbf{c}' = \frac{1}{3}(\mathbf{a}+\mathbf{b}+\mathbf{c})$ $P\bar{3}1c$ (163);
[2] $\mathbf{a}' = \frac{1}{2}(-\mathbf{a}+\mathbf{b}+\mathbf{c}),\ \mathbf{b}' = \frac{1}{2}(\mathbf{a}-\mathbf{b}+\mathbf{c}),\ \mathbf{c}' = \frac{1}{2}(\mathbf{a}+\mathbf{b}-\mathbf{c})$ $R\bar{3}m$ (166)