International Tables for Crystallography (2006). Vol. A1, Maximal subgroups of space group 228, pp. 390–392.

 $F4_{1}/d\bar{3}2/c$

 D_{h}^{8}

$Fd\bar{3}c$

ORIGIN CHOICE 1, Origin at 23, at $-\frac{3}{8}, -\frac{3}{8}, -\frac{3}{8}$ from centre ($\overline{3}$)

Generators selected (1); t(1,0,0); t(0,1,0); t(0,0,1); $t(0,\frac{1}{2},\frac{1}{2})$; $t(\frac{1}{2},0,\frac{1}{2})$; (2); (3); (5); (13); (25)

No. 228

General position

Wyckoff letter, Site symmetry $(0,0,0)+(0,\frac{1}{2},\frac{1}{2})+(\frac{1}{2},0,\frac{1}{2})+(\frac{1}{2},\frac{1}{2},0)+$	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$-\frac{3}{4}$ - $\frac{1}{4}$ - $\frac{1}{4}$ - $\frac{3}{4}$ - $\frac{1}{4}$

I Maximal translationengleiche subgroups

$[2] F\bar{4}3c$ (219)	(1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; 37; 38; 39;		
	40; 41; 42; 43; 44; 45; 46; 47; 48)+		
$[2] F4_132$ (210)	(1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; 13; 14; 15;		
	16; 17; 18; 19; 20; 21; 22; 23; 24) +		
$[2] Fd\bar{3}1 (203, Fd\bar{3})$	(1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; 25; 26; 27;		1/4, 1/4, 1/4
	28; 29; 30; 31; 32; 33; 34; 35; 36)+		
$\left([3] F4_1/d12/c (142, I4_1/acd) \right)$	(1; 2; 3; 4; 13; 14; 15; 16; 25; 26; 27; 28; 37;	$1/2({\bf a}-{\bf b}), 1/2({\bf a}+{\bf b}), {\bf c}$	0, 0, 1/4
	38; 39; 40)+		
$\int [3] F4_1/d12/c (142, I4_1/acd)$	(1; 4; 2; 3; 18; 19; 17; 20; 25; 28; 26; 27; 42;	$1/2({\bf b}-{\bf c}), 1/2({\bf b}+{\bf c}), {\bf a}$	1/4, 0, 0
	43; 41; 44)+		
[3] $F4_1/d12/c$ (142, $I4_1/acd$)	(1; 3; 4; 2; 22; 24; 23; 21; 25; 27; 28; 26; 46;	1/2(-a+c), 1/2(a+c), b	0, 1/4, 0
l	48; 47; 45)+		
$\int [4] F 1\bar{3}2/c \ (167, R\bar{3}c)$	(1; 5; 9; 14; 19; 24; 25; 29; 33; 38; 43; 48) +	1/2(-a+b), 1/2(-b+c), a+	$\mathbf{b} + \mathbf{c}$
			3/8,3/8,3/8
[4] $F1\bar{3}2/c$ (167, $R\bar{3}c$)	(1; 6; 12; 13; 18; 24; 25; 30; 36; 37; 42; 48) +	1/2(a+b), 1/2(-b-c), -a+	b – c
			1/8,3/8,1/8
$\int [4] F 1\bar{3}2/c \ (167, R\bar{3}c)$	(1; 7; 10; 13; 19; 22; 25; 31; 34; 37; 43; 46) +	1/2(-a-b), 1/2(b-c), a-b	- c
	(· · · · · · · · · · · · · · · · · · ·		3/8,1/8,1/8
[4] $F1\bar{3}2/c$ (167, $R\bar{3}c$)	(1; 8; 11; 14; 18; 22; 25; 32; 35; 38; 42; 46) +	1/2(a-b), 1/2(b+c), -a-b	
			1/8,1/8,3/8
`			, , , , , ,

II Maximal klassengleiche subgroups

• Loss of centring translations	none
• Enlarged unit cell	none

No. 228

• Series of maximal isomorphic subgroups

$[p^3]$ a ' = p a , b ' = p b , c ' = p c			
$Fd\bar{3}c$ (228)	$(2+(\frac{1}{2}+2u,\frac{p}{2}+2v,\frac{p}{2}-\frac{1}{2}); 3+(\frac{p}{2}+2u,\frac{p}{2}-\frac{1}{2},\frac{1}{2}+2w);$	p a , p b , p c	1/4 + u, 1/4 + v, 1/4 + w
	5 + (u - w, -u + v, -v + w);		
	$13 + (\frac{3p}{4} - \frac{3}{4} + u - v, \frac{p}{4} - \frac{1}{4} - u + v, \frac{3p}{4} - \frac{1}{4} + 2w);$		
	$25 + \left(\frac{3p}{4} - \frac{1}{4} + 2u, \frac{3p}{4} - \frac{1}{4} + 2v, \frac{3p}{4} - \frac{1}{4} + 2w\right)\rangle$		
	$p > 2; 0 \le u < p; 0 \le v < p; 0 \le w < p$		
	p^3 conjugate subgroups for prime $p \equiv 3 \pmod{4}$		
$Fd\bar{3}c$ (228)	$\langle 2 + (2u, \frac{p}{2} - \frac{1}{2} + 2v, \frac{p}{2} - \frac{1}{2}); 3 + (\frac{p}{2} - \frac{1}{2} + 2u, \frac{p}{2} - \frac{1}{2}, 2w);$	p a , p b , p c	u, v, w
	5 + (u - w, -u + v, -v + w);		
	$13 + (\frac{3p}{4} - \frac{3}{4} + u - v, \frac{p}{4} - \frac{1}{4} - u + v, \frac{3p}{4} - \frac{3}{4} + 2w);$		
	$25 + \left(\frac{3p}{4} - \frac{3}{4} + 2u, \frac{3p}{4} - \frac{3}{4} + 2v, \frac{3p}{4} - \frac{3}{4} + 2w\right)\rangle$		
	$p > 4; \ 0 \le u < p; \ 0 \le v < p; \ 0 \le w < p$		
	p^3 conjugate subgroups for prime $p \equiv 1 \pmod{4}$		

I Minimal *translationengleiche* supergroups

II Minimal non-isomorphic klassengleiche supergroups

• Additional centring translations

• Decreased unit cell

[2] $\mathbf{a}' = \frac{1}{2}\mathbf{a}, \ \mathbf{b}' = \frac{1}{2}\mathbf{b}, \ \mathbf{c}' = \frac{1}{2}\mathbf{c} \ Pn\bar{3}m$ (224)

(Continued from the following page)

• Series of maximal isomorphic subgroups

$[p^3] \mathbf{a}' = p\mathbf{a}, \ \mathbf{b}' = p\mathbf{b}, \ \mathbf{c}' = p\mathbf{c}$ $Fd\bar{3}c$ (228)	$\langle 2 + (\frac{p}{4} - \frac{1}{4} + 2u, \frac{3p}{4} - \frac{3}{4} + 2v, \frac{p}{2} - \frac{1}{2});$	p a , p b , p c
	$3 + (\frac{3p}{4} - \frac{3}{4} + 2u, \frac{p}{2} - \frac{1}{2}, \frac{p}{4} - \frac{1}{4} + 2w);$ 5 + (u - w, -u + v, -v + w); $13 + (\frac{3p}{4} - \frac{3}{4} + u - v, \frac{p}{4} - \frac{1}{4} - u + v, 2w);$ $25 + (2u, 2v, 2w)\rangle$ p > 2; $0 \le u < p;$ $0 \le v < p;$ p^{3} conjugate subgroups for the prime p	

No. 228

I Minimal translationengleiche supergroups

II Minimal non-isomorphic klassengleiche supergroups

• Additional centring translations

• Decreased unit cell

[2] $\mathbf{a}' = \frac{1}{2}\mathbf{a}, \ \mathbf{b}' = \frac{1}{2}\mathbf{b}, \ \mathbf{c}' = \frac{1}{2}\mathbf{c} \ Pn\bar{3}m$ (224)

u, v, w

Origin choice 2 $F d \bar{3} c$

none

none

none

none

$Fd\bar{3}c$

ORIGIN CHOICE 2, Origin at centre $(\bar{3})$, at $\frac{3}{8}, \frac{3}{8}, \frac{3}{8}$ from 23

Generators selected (1); t(1,0,0); t(0,1,0); t(0,0,1); $t(0,\frac{1}{2},\frac{1}{2})$; $t(\frac{1}{2},0,\frac{1}{2})$; (2); (3); (5); (13); (25)

General position

Multiplicity,		Coor	rdinates	
Wyckoff letter, Site symmetry		$(0,0,0)+$ $(0,\frac{1}{2},\frac{1}{2})+$	$(\frac{1}{2},0,\frac{1}{2})+(\frac{1}{2},\frac{1}{2},0)+$	
192 h 1		$\begin{array}{c} (2) \ \bar{x} + \frac{1}{4}, \bar{y} + \frac{3}{4}, z + \frac{1}{2} \\ (6) \ z + \frac{1}{2}, \bar{x} + \frac{1}{4}, \bar{y} + \frac{3}{4} \\ (10) \ \bar{y} + \frac{3}{4}, z + \frac{1}{2}, \bar{x} + \frac{1}{4} \\ (14) \ \bar{y} + \frac{1}{2}, \bar{x} + \frac{1}{2}, \bar{z} + \frac{1}{2} \\ (18) \ \bar{x}, z + \frac{3}{4}, y + \frac{1}{4} \\ (22) \ z + \frac{1}{4}, \bar{y}, x + \frac{3}{4} \\ (26) \ x + \frac{3}{4}, y + \frac{1}{4}, \bar{z} + \frac{1}{2} \\ (30) \ \bar{z} + \frac{1}{2}, x + \frac{3}{4}, y + \frac{1}{4} \\ (34) \ y + \frac{1}{4}, \bar{z} + \frac{1}{2}, x + \frac{3}{4} \\ (38) \ y + \frac{1}{2}, x + \frac{1}{2}, z + \frac{1}{2} \\ (42) \ x, \bar{z} + \frac{1}{4}, \bar{y} + \frac{3}{4} \end{array}$	$(3) \ \bar{x} + \frac{3}{4}, y + \frac{1}{2}, \bar{z} + \frac{1}{4} (7) \ \bar{z} + \frac{1}{4}, \bar{x} + \frac{3}{4}, y + \frac{1}{2} (11) \ y + \frac{1}{2}, \bar{z} + \frac{1}{4}, \bar{x} + \frac{3}{4} (15) \ y + \frac{1}{4}, \bar{x}, z + \frac{3}{4} (19) \ \bar{x} + \frac{1}{2}, \bar{z} + \frac{1}{2}, \bar{y} + \frac{1}{2} (23) \ \bar{z}, y + \frac{3}{4}, x + \frac{1}{4} (27) \ x + \frac{1}{4}, \bar{y} + \frac{1}{2}, z + \frac{3}{4} (31) \ z + \frac{3}{4}, x + \frac{1}{4}, \bar{y} + \frac{1}{2} (35) \ \bar{y} + \frac{1}{2}, z + \frac{3}{4}, x + \frac{1}{4} (39) \ \bar{y} + \frac{3}{4}, x, \bar{z} + \frac{1}{4} $	$(4) x + \frac{1}{2}, \bar{y} + \frac{1}{4}, \bar{z} + \frac{3}{4} \\(8) \bar{z} + \frac{3}{4}, x + \frac{1}{2}, \bar{y} + \frac{1}{4} \\(12) \bar{y} + \frac{1}{4}, \bar{z} + \frac{3}{4}, x + \frac{1}{2} \\(16) \bar{y}, x + \frac{3}{4}, z + \frac{1}{4} \\(20) x + \frac{1}{4}, \bar{z}, y + \frac{3}{4} \\(24) \bar{z} + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{x} + \frac{1}{2} \\(28) \bar{x} + \frac{1}{2}, y + \frac{3}{4}, z + \frac{1}{4} \\(32) z + \frac{1}{4}, \bar{x} + \frac{1}{2}, y + \frac{3}{4} \\(36) y + \frac{3}{4}, z + \frac{1}{4}, \bar{x} + \frac{1}{2} \\(40) y, \bar{x} + \frac{1}{4}, \bar{z} + \frac{3}{4} \\(44) \bar{z} + \frac{3}{4}, \bar{z} + \frac{1}{4} \\(44) \bar{z} + \frac{3}{4} \\(44) \bar{z} + \frac{3}{4}, \bar{z} + \frac{1}{4} \\(44) \bar{z} + \frac{3}{4} \\(44) \bar{z} + \frac$
	(41) $\bar{x} + \frac{1}{4}, \bar{z} + \frac{3}{4}, y$ (45) $\bar{z} + \frac{1}{4}, \bar{y} + \frac{3}{4}, x$	(42) $x, z + \frac{1}{4}, y + \frac{1}{4}$ (46) $\overline{z} + \frac{3}{4}, y, \overline{x} + \frac{1}{4}$	(43) $x + \frac{1}{2}, z + \frac{1}{2}, y + \frac{1}{2}$ (47) $z, \bar{y} + \frac{1}{4}, \bar{x} + \frac{3}{4}$	(44) $\bar{x} + \frac{3}{4}, z, \bar{y} + \frac{1}{4}$ (48) $z + \frac{1}{2}, y + \frac{1}{2}, x + \frac{1}{2}$

I Maximal translationengleiche subgroups

	$[2] F\bar{4}3c$ (219)	(1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; 37; 38; 39;	1/8, 1/8, 1/8
	[2] <i>F</i> 4 ₁ 32 (210)	40; 41; 42; 43; 44; 45; 46; 47; 48)+ (1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; 13; 14; 15; 16; 17; 18; 19; 20; 21; 22; 23; 24)+	1/8, 1/8, 1/8
	[2] $Fd\bar{3}1$ (203, $Fd\bar{3}$)	(1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; 25; 26; 27; 28; 29; 30; 31; 32; 33; 34; 35; 36)+	
ſ	$[3] F4_1/d12/c (142, I4_1/acd)$	(1; 2; 3; 4; 13; 14; 15; 16; 25; 26; 27; 28; 37; 38; 39; 40)+	1/2(a-b), 1/2(a+b), c
ł	[3] $F4_1/d12/c$ (142, $I4_1/acd$)	(1; 4; 2; 3; 18; 19; 17; 20; 25; 28; 26; 27; 42; 43; 41; 44)+	$1/2({\bf b}-{\bf c}), 1/2({\bf b}+{\bf c}), {\bf a}$
	[3] $F4_1/d12/c$ (142, $I4_1/acd$)	(1; 3; 4; 2; 22; 24; 23; 21; 25; 27; 28; 26; 46; 48; 47; 45)+	1/2(-a+c), 1/2(a+c), b
Ì	$(4] F1\bar{3}2/c (167, R\bar{3}c)$	(1; 5; 9; 14; 19; 24; 25; 29; 33; 38; 43; 48)+	1/2(-a+b), 1/2(-b+c), a+b+c
	[4] $F1\bar{3}2/c$ (167, $R\bar{3}c$)	(1; 6; 12; 13; 18; 24; 25; 30; 36; 37; 42; 48)+	$1/2(\mathbf{a}+\mathbf{b}), 1/2(-\mathbf{b}-\mathbf{c}), -\mathbf{a}+\mathbf{b}-\mathbf{c}$ 1/4,0,1/4
ł	[4] $F1\bar{3}2/c$ (167, $R\bar{3}c$)	(1; 7; 10; 13; 19; 22; 25; 31; 34; 37; 43; 46)+	$1/2(-\mathbf{a}-\mathbf{b}), 1/2(\mathbf{b}-\mathbf{c}), \mathbf{a}-\mathbf{b}-\mathbf{c}$ 0, 1/4, 1/4
	[4] $F1\bar{3}2/c$ (167, $R\bar{3}c$)	(1; 8; 11; 14; 18; 22; 25; 32; 35; 38; 42; 46)+	$1/2(\mathbf{a}-\mathbf{b}), 1/2(\mathbf{b}+\mathbf{c}), -\mathbf{a}-\mathbf{b}+\mathbf{c}$ 1/4, 1/4, 0

II Maximal klassengleiche subgroups

Loss of centring translations Enlarged unit cell

none none

(*Continued on the preceding page*)

No. 228

• Series of maximal isomorphic subgroups

$[p^3]$ a ' = p a , b ' = p b , c ' = p c			
$Fd\bar{3}c$ (228)	$(2+(\frac{1}{2}+2u,\frac{p}{2}+2v,\frac{p}{2}-\frac{1}{2}); 3+(\frac{p}{2}+2u,\frac{p}{2}-\frac{1}{2},\frac{1}{2}+2w);$	p a , p b , p c	1/4 + u, 1/4 + v, 1/4 + w
	5 + (u - w, -u + v, -v + w);		
	$13 + (\frac{3p}{4} - \frac{3}{4} + u - v, \frac{p}{4} - \frac{1}{4} - u + v, \frac{3p}{4} - \frac{1}{4} + 2w);$		
	$25 + \left(\frac{3p}{4} - \frac{1}{4} + 2u, \frac{3p}{4} - \frac{1}{4} + 2v, \frac{3p}{4} - \frac{1}{4} + 2w\right)\rangle$		
	$p > 2; 0 \le u < p; 0 \le v < p; 0 \le w < p$		
	p^3 conjugate subgroups for prime $p \equiv 3 \pmod{4}$		
$Fd\bar{3}c$ (228)	$\langle 2 + (2u, \frac{p}{2} - \frac{1}{2} + 2v, \frac{p}{2} - \frac{1}{2}); 3 + (\frac{p}{2} - \frac{1}{2} + 2u, \frac{p}{2} - \frac{1}{2}, 2w);$	p a , p b , p c	u, v, w
	5 + (u - w, -u + v, -v + w);		
	$13 + (\frac{3p}{4} - \frac{3}{4} + u - v, \frac{p}{4} - \frac{1}{4} - u + v, \frac{3p}{4} - \frac{3}{4} + 2w);$		
	$25 + \left(\frac{3p}{4} - \frac{3}{4} + 2u, \frac{3p}{4} - \frac{3}{4} + 2v, \frac{3p}{4} - \frac{3}{4} + 2w\right)\rangle$		
	$p > 4; \ 0 \le u < p; \ 0 \le v < p; \ 0 \le w < p$		
	p^3 conjugate subgroups for prime $p \equiv 1 \pmod{4}$		

I Minimal *translationengleiche* supergroups

II Minimal non-isomorphic klassengleiche supergroups

• Additional centring translations

• Decreased unit cell

[2] $\mathbf{a}' = \frac{1}{2}\mathbf{a}, \ \mathbf{b}' = \frac{1}{2}\mathbf{b}, \ \mathbf{c}' = \frac{1}{2}\mathbf{c} \ Pn\bar{3}m$ (224)

(Continued from the following page)

• Series of maximal isomorphic subgroups

$[p^3] \mathbf{a}' = p\mathbf{a}, \ \mathbf{b}' = p\mathbf{b}, \ \mathbf{c}' = p\mathbf{c}$ $Fd\bar{3}c$ (228)	$\langle 2 + (\frac{p}{4} - \frac{1}{4} + 2u, \frac{3p}{4} - \frac{3}{4} + 2v, \frac{p}{2} - \frac{1}{2});$	p a , p b , p c
	$3 + (\frac{3p}{4} - \frac{3}{4} + 2u, \frac{p}{2} - \frac{1}{2}, \frac{p}{4} - \frac{1}{4} + 2w);$ 5 + (u - w, -u + v, -v + w); $13 + (\frac{3p}{4} - \frac{3}{4} + u - v, \frac{p}{4} - \frac{1}{4} - u + v, 2w);$ $25 + (2u, 2v, 2w)\rangle$ p > 2; $0 \le u < p;$ $0 \le v < p;$ p^{3} conjugate subgroups for the prime p	

No. 228

I Minimal translationengleiche supergroups

II Minimal non-isomorphic klassengleiche supergroups

• Additional centring translations

• Decreased unit cell

[2] $\mathbf{a}' = \frac{1}{2}\mathbf{a}, \ \mathbf{b}' = \frac{1}{2}\mathbf{b}, \ \mathbf{c}' = \frac{1}{2}\mathbf{c} \ Pn\bar{3}m$ (224)

u, v, w

Origin choice 2 $F d \bar{3} c$

none

none

none

none