

D_3^7

HEXAGONAL AXES

No. 155

 $R32$

| Axes | | Coordinates | Wyckoff positions | | | | | |
|---|-------------------|--|---|---|---------------------------------------|---|---|------------------|
| | | | $3a$ | $3b$ | $6c$ | $9d$ | $9e$ | $18f$ |
| I Maximal translationengleiche subgroups | | | | | | | | |
| [2] | $R3$ (146) | | $3a$ | $3a$ | $2 \times 3a$ | $9b$ | $9b$ | $2 \times 9b$ |
| [3] | C_{121} (5) | $\frac{1}{3}(2\mathbf{a}+\mathbf{b}-2\mathbf{c}), \mathbf{b},$ $\frac{1}{3}(2\mathbf{a}+\mathbf{b}+\mathbf{c})$ | $2a$ | $2b$ | $4c$ | $2a; 4c$ | $2b; 4c$ | $3 \times 4c$ |
| | conjugate: | $\frac{1}{3}(-\mathbf{a}+\mathbf{b}-2\mathbf{c}),$ $-\mathbf{a}-\mathbf{b}, \frac{1}{3}(-\mathbf{a}+\mathbf{b}+\mathbf{c})$ | | | | | | |
| | conjugate: | $\frac{1}{3}(-\mathbf{a}-2\mathbf{b}-2\mathbf{c}), \mathbf{a},$ $\frac{1}{3}(-\mathbf{a}-2\mathbf{b}+\mathbf{c})$ | | | | | | |
| | alternative: | $\frac{1}{3}(2\mathbf{a}+\mathbf{b}-2\mathbf{c}), \mathbf{b}, \mathbf{c}$ | $\frac{3}{2}x, -\frac{1}{2}x+y, x+z$ | | | | | |
| | or | $\frac{1}{3}(-\mathbf{a}+\mathbf{b}-2\mathbf{c}),$ $-\mathbf{a}-\mathbf{b}, \mathbf{c}$ | $\frac{3}{2}(-x+y), -\frac{1}{2}(x+y),$ $-x+y+z$ | | | | | |
| | or | $\frac{1}{3}(-\mathbf{a}-2\mathbf{b}-2\mathbf{c}), \mathbf{a}, \mathbf{c}$ | $-\frac{3}{2}y, x-\frac{1}{2}y, -y+z$ | | | | | |
| | alternative: | $2\mathbf{a}+\mathbf{b}, -\mathbf{b},$ $-\frac{1}{3}(2\mathbf{a}+\mathbf{b}+\mathbf{c})$ | $\frac{1}{2}x-z, \frac{1}{2}x-y, -3z$ | | | | | |
| | or | $-\mathbf{a}+\mathbf{b}, \mathbf{a}+\mathbf{b},$ $\frac{1}{3}(\mathbf{a}-\mathbf{b}-\mathbf{c})$ | $\frac{1}{2}(-x+y)-z, \frac{1}{2}(x+y), -3z$ | | | | | |
| | or | $-\mathbf{a}-2\mathbf{b}, -\mathbf{a},$ $\frac{1}{3}(\mathbf{a}+2\mathbf{b}-\mathbf{c})$ | $-\frac{1}{2}y-z, -x+\frac{1}{2}y, -3z$ | | | | | |
| II Maximal klassengleiche subgroups | | | | | | | | |
| Loss of centring translations | | | | | | | | |
| [3] | $P3_221$ (154) | $x+\frac{1}{3}, y+\frac{1}{3}, z$ 3 conjugate subgroups | $3a$ | $3b$ | $6c$ | $3a; 6c$ | $3b; 6c$ | $3 \times 6c$ |
| [3] | $P3_121$ (152) | $x-\frac{1}{3}, y-\frac{1}{3}, z$ 3 conjugate subgroups | $3a$ | $3b$ | $6c$ | $3a; 6c$ | $3b; 6c$ | $3 \times 6c$ |
| [3] | $P321$ (150) | 3 conjugate subgroups | $1a; 2d$ | $1b; 2d$ | $2c; 2 \times 2d$ | $3e; 6g$ | $3f; 6g$ | $3 \times 6g$ |
| Enlarged unit cell, isomorphic | | | | | | | | |
| [2] | $R32$ | $-\mathbf{a}, -\mathbf{b}, 2\mathbf{c}$ $-x, -y, \frac{1}{2}z; +(0, 0, \frac{1}{2})$ | $3a; 3b$ | $6c$ | $2 \times 6c$ | $9d; 9e$ | $18f$ | $2 \times 18f$ |
| [2] | $R32$ | $-\mathbf{a}, -\mathbf{b}, 2\mathbf{c}$ $-x, -y, \frac{1}{2}z+\frac{1}{4}; +(0, 0, \frac{1}{2})$ | $6c$ | $3a; 3b$ | $2 \times 6c$ | $18f$ | $9d; 9e$ | $2 \times 18f$ |
| [5] | $R32$ | $-\mathbf{a}, -\mathbf{b}, 5\mathbf{c}$ $-x, -y, \frac{1}{5}z; \pm(0, 0, \frac{1}{5})$ $\pm(0, 0, \frac{2}{5})$ | $3a; 2 \times 6c$ | $3b; 2 \times 6c$ | $5 \times 6c$ | $9d; 2 \times 18f$ | $9e; 2 \times 18f$ | $5 \times 18f$ |
| [p] | $R32$ | $-\mathbf{a}, -\mathbf{b}, p\mathbf{c}$ $-x, -y, \frac{1}{p}z; +(0, 0, \frac{u}{p})$ $p = \text{prime} = 6n-1; u = 1, \dots, p-1$ | $3a; \frac{p-1}{2} \times 6c$ | $3b; \frac{p-1}{2} \times 6c$ | $p \times 6c$ | $9d; \frac{p-1}{2} \times 18f$ | $9e; \frac{p-1}{2} \times 18f$ | $p \times 18f$ |
| | | $\mathbf{a}, \mathbf{b}, p\mathbf{c}$ $x, y, \frac{1}{p}z; +(0, 0, \frac{u}{p})$ $p = \text{prime} = 6n+1; u = 1, \dots, p-1$ | | | | | | |
| [4] | $R32$ | $-2\mathbf{a}, -2\mathbf{b}, \mathbf{c}$ $-\frac{1}{2}x, -\frac{1}{2}y, z; +(\frac{1}{2}, 0, 0);$ $+(0, \frac{1}{2}, 0); +(\frac{1}{2}, \frac{1}{2}, 0)$ | $3a; 9d$ | $3b; 9e$ | $6c; 18f$ | $2 \times 9d; 18f$ | $2 \times 9e; 18f$ | $4 \times 18f$ |
| [p^2] | $R32$ | $-p\mathbf{a}, -p\mathbf{b}, \mathbf{c}$ $-\frac{1}{p}x, -\frac{1}{p}y, z; +(\frac{u}{p}, \frac{v}{p}, 0)$ $p = \text{prime} = 6n-1; u, v = 1, \dots, p-1$ | $3a; (p-1) \times 9d;$ $\frac{(p-1)(p-2)}{6} \times 18f$ | $3b; (p-1) \times 9e;$ $\frac{(p-1)(p-2)}{6} \times 18f$ | $6c;$ $\frac{p^2-1}{3} \times 18f$ | $p \times 9d;$ $\frac{p(p-1)}{2} \times 18f$ | $p \times 9e;$ $\frac{p(p-1)}{2} \times 18f$ | $p^2 \times 18f$ |
| | | $p\mathbf{a}, p\mathbf{b}, \mathbf{c}$ $\frac{1}{p}x, \frac{1}{p}y, z; +(\frac{u}{p}, \frac{v}{p}, 0)$ $p = \text{prime} = 6n+1; u, v = 1, \dots, p-1$ | | | | | | |