

$P\bar{6}$ 

No. 174

 $C_{3h}^1$ 

Axes

Coordinates

	$1a$	$1b$	$1c$ $2h$	$1d$ $2i$	$1e$ $3j$	$1f$ $3k$	$2g$ $6l$	
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**I Maximal translationengleiche subgroups**[2]  $P3$  (143)

$1a$	$1a$	$1b$ $2 \times 1b$	$1b$ $2 \times 1c$	$1c$ $3d$	$1c$ $3d$	$2 \times 1a$ $2 \times 3d$	
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[3]  $P11m$  (6)

$1a$	$1b$	$1a$ $2c$	$1b$ $2c$	$1a$ $3 \times 1a$	$1b$ $3 \times 1b$	$2c$ $3 \times 2c$	
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**II Maximal klassengleiche subgroups**

Enlarged unit cell, isomorphic

[2] $P\bar{6}$	$\mathbf{a}, \mathbf{b}, 2\mathbf{c}$	$x, y, \frac{1}{2}z; + (0, 0, \frac{1}{2})$	$1a; 1b$	$2g$	$1c; 1d$ $2h$	$2h$ $2 \times 2i$	$1e; 1f$ $3j; 3k$	$2i$ $6l$	$2 \times 2g$ $2 \times 6l$
[2] $P\bar{6}$	$\mathbf{a}, \mathbf{b}, 2\mathbf{c}$	$x, y, \frac{1}{2}z + \frac{1}{4}; + (0, 0, \frac{1}{2})$	$2g$	$1a; 1b$	$2h$ $2 \times 2h$	$1c; 1d$ $2 \times 2i$	$2i$ $6l$	$1e; 1f$ $3j; 3k$	$2 \times 2g$ $2 \times 6l$
[3] $P\bar{6}$	$\mathbf{a}, \mathbf{b}, 3\mathbf{c}$	$x, y, \frac{1}{3}z; \pm (0, 0, \frac{1}{3})$	$1a; 2g$	$1b; 2g$	$1c; 2h$ $3 \times 2h$	$1d; 2h$ $3 \times 2i$	$1e; 2i$ $3j; 6l$	$1f; 2i$ $3k; 6l$	$3 \times 2g$ $3 \times 6l$
[p] $P\bar{6}$	$\mathbf{a}, \mathbf{b}, p\mathbf{c}$ $p = \text{prime} > 2; u = 1, \dots, p-1$	$x, y, \frac{1}{p}z; + (0, 0, \frac{u}{p})$	$1a; \frac{p-1}{2} \times 2g$	$1b; \frac{p-1}{2} \times 2g$	$1c; \frac{p-1}{2} \times 2h$ $p \times 2h$	$1d; \frac{p-1}{2} \times 2h$ $p \times 2i$	$1e; \frac{p-1}{2} \times 2i$ $3j; \frac{p-1}{2} \times 6l$	$1f; \frac{p-1}{2} \times 2i$ $3k; \frac{p-1}{2} \times 6l$	$p \times 2g$ $p \times 6l$
[3] $P\bar{6}$	$2\mathbf{a}+\mathbf{b}, -\mathbf{a}+\mathbf{b}, \mathbf{c}$	$\frac{1}{3}(x+y), \frac{1}{3}(-x+2y), z; \pm (\frac{1}{3}, \frac{2}{3}, 0)$	$1a; 1c; 1e$	$1b; 1d; 1f$	$3j$ $6l$	$3k$ $6l$	$3j$ $3 \times 3j$	$3k$ $3 \times 3k$	$2g; 2h; 2i$ $3 \times 6l$
[3] $P\bar{6}$	$2\mathbf{a}+\mathbf{b}, -\mathbf{a}+\mathbf{b}, \mathbf{c}$	$\frac{1}{3}(x+y) + \frac{1}{3}, \frac{1}{3}(-x+2y), z; \pm (\frac{1}{3}, \frac{2}{3}, 0)$	$3j$	$3k$	$1a; 1c; 1e$ $2g; 2h; 2i$	$1b; 1d; 1f$ $6l$	$3j$ $3 \times 3j$	$3k$ $3 \times 3k$	$6l$ $3 \times 6l$
[3] $P\bar{6}$	$2\mathbf{a}+\mathbf{b}, -\mathbf{a}+\mathbf{b}, \mathbf{c}$	$\frac{1}{3}(x+y) - \frac{1}{3}, \frac{1}{3}(-x+2y), z; \pm (\frac{1}{3}, \frac{2}{3}, 0)$	$3j$	$3k$	$3j$ $6l$	$3k$ $2g; 2h; 2i$	$1a; 1c; 1e$ $3 \times 3j$	$1b; 1d; 1f$ $3 \times 3k$	$6l$ $3 \times 6l$
[7] $P\bar{6}$	$3\mathbf{a}+\mathbf{b}, -\mathbf{a}+2\mathbf{b}, \mathbf{c}$	$\frac{1}{7}(2x+y), \frac{1}{7}(-x+3y), z; \pm (\frac{1}{7}, \frac{3}{7}, 0); \pm (\frac{3}{7}, \frac{2}{7}, 0); \pm (\frac{5}{7}, \frac{1}{7}, 0)$	$1a; 2 \times 3j$	$1b; 2 \times 3k$	$1c; 2 \times 3j$ $2h; 2 \times 6l$	$1d; 2 \times 3k$ $2i; 2 \times 6l$	$1e; 2 \times 3j$ $p \times 3j$	$1f; 2 \times 3k$ $p \times 3k$	$2g; 2 \times 6l$ $p \times 6l$
[7] $P\bar{6}$	$3\mathbf{a}+2\mathbf{b}, -2\mathbf{a}+\mathbf{b}, \mathbf{c}$	$\frac{1}{7}(x+2y), \frac{1}{7}(-2x+3y), z; \pm (\frac{2}{7}, \frac{3}{7}, 0); \pm (\frac{3}{7}, \frac{1}{7}, 0); \pm (\frac{1}{7}, \frac{5}{7}, 0)$	$1a; 2 \times 3j$	$1b; 2 \times 3k$	$1e; 2 \times 3j$ $2i; 2 \times 6l$	$1f; 2 \times 3k$ $2h; 2 \times 6l$	$1c; 2 \times 3j$ $p \times 3j$	$1d; 2 \times 3k$ $p \times 3k$	$2g; 2 \times 6l$ $p \times 6l$
[p] $P\bar{6}$	$q\mathbf{a}+r\mathbf{b}, -r\mathbf{a}+(q-r)\mathbf{b}, \mathbf{c}$ $p = \text{prime} = q^2 - qr + r^2 = 6n + 1;$ $q, r = 1, 2, \dots; q > r; u = 1, \dots, p-1$	$\frac{1}{p}((q-r)x+ry), \frac{1}{p}(-rx+qy), z; + (\frac{ur}{p}, \frac{uq}{p}, 0)$	$1a; \frac{p-1}{3} \times 3j$	$1b; \frac{p-1}{3} \times 3k$	$1c(e^*); \frac{p-1}{3} \times 3j$ $2h(i^*); \frac{p-1}{3} \times 6l$	$1d(f^*); \frac{p-1}{3} \times 3k$ $2i(h^*); \frac{p-1}{3} \times 6l$	$1e(c^*); \frac{p-1}{3} \times 3j$ $2i(h^*); \frac{p-1}{3} \times 6l$	$1f(d^*); \frac{p-1}{3} \times 3k$ $p \times 3j$	$2g; p \times 6l$ $p \times 6l$
[4] $P\bar{6}$	$2\mathbf{a}, 2\mathbf{b}, \mathbf{c}$	$\frac{1}{2}x, \frac{1}{2}y, z; + (\frac{1}{2}, 0, 0); + (0, \frac{1}{2}, 0); + (\frac{1}{2}, \frac{1}{2}, 0)$	$1a; 3j$	$1b; 3k$	$1e; 3j$ $2i; 6l$	$1f; 3k$ $2h; 6l$	$1c; 3j$ $4 \times 3j$	$1d; 3k$ $4 \times 3k$	$2g; 6l$ $4 \times 6l$
[ $p^2$ ] $P\bar{6}$	$p\mathbf{a}, p\mathbf{b}, \mathbf{c}$ $p = \text{prime} = 3n - 1; u, v = 1, \dots, p-1$	$\frac{1}{p}x, \frac{1}{p}y, z; + (\frac{u}{p}, \frac{v}{p}, 0)$	$1a; \frac{p^2-1}{3} \times 3j$	$1b; \frac{p^2-1}{3} \times 3k$	$1e; \frac{p^2-1}{3} \times 3j$ $2i; \frac{p^2-1}{3} \times 6l$	$1f; \frac{p^2-1}{3} \times 3k$ $2h; \frac{p^2-1}{3} \times 6l$	$1c; \frac{p^2-1}{3} \times 3j$ $p^2 \times 3j$	$1d; \frac{p^2-1}{3} \times 3k$ $p^2 \times 3k$	$2g; \frac{p^2-1}{3} \times 6l$ $p^2 \times 6l$

$$* q+r = 3n-1$$