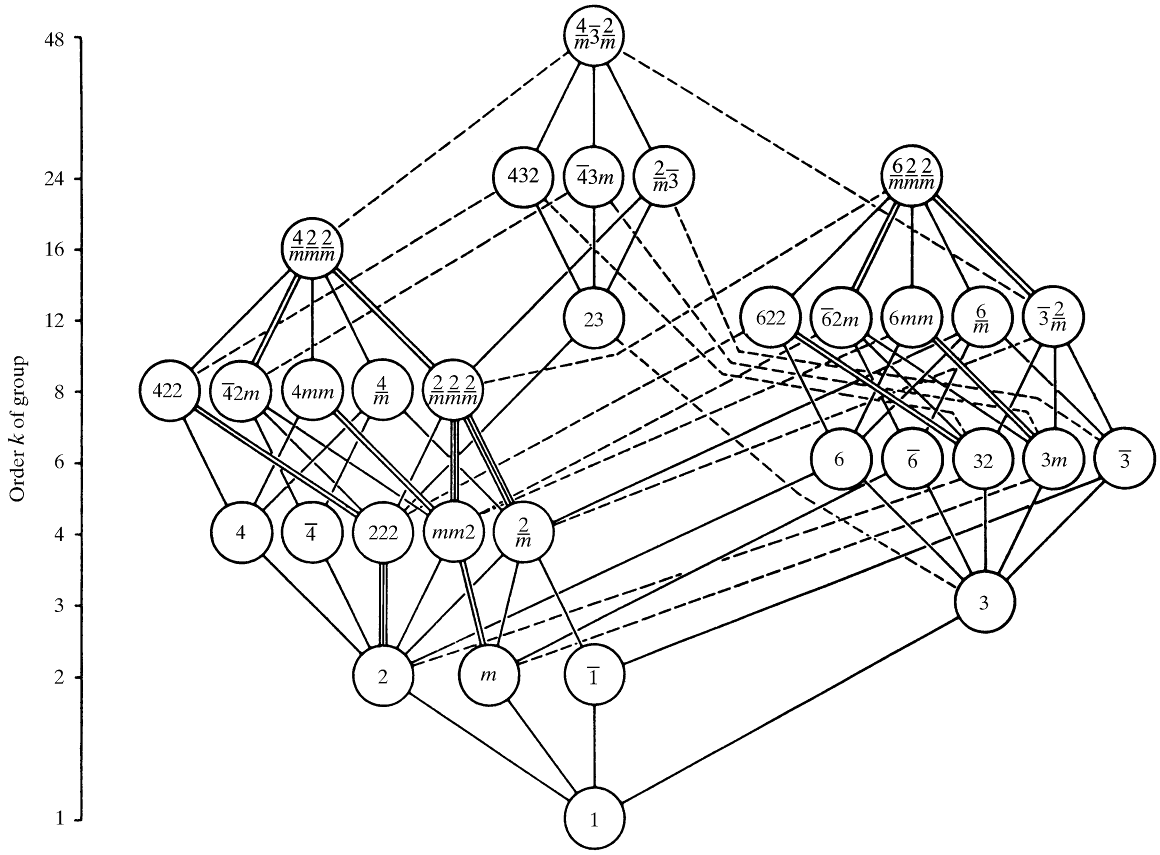
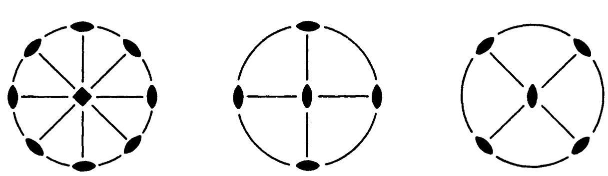


Fig. 10.1.3.2. Maximal subgroups and minimal supergroups of the three-dimensional crystallographic point groups. Solid lines indicate maximal normal subgroups; double or triple solid lines mean that there are two or three maximal normal subgroups with the same symbol. Dashed lines refer to sets of maximal conjugate subgroups. The group orders are given on the left. Full Hermann–Mauguin symbols are used.

symbols' 222. and 2.22.



422 222. 2.22

(2) Similarly, group 432 has one maximal normal subgroup, 23.

Figs. 10.1.3.1 and 10.1.3.2 show that there exist two 'summits' in both two and three dimensions from which all other point groups can be derived by 'chains' of maximal subgroups. These summits are formed by the square and the hexagonal holohedry in two dimensions and by the cubic and the hexagonal holohedry in three dimensions.

The sub- and supergroups of the point groups are useful both in their own right and as basis of the *translationengleiche* or *t* subgroups and supergroups of space groups; this is set out in Section 2.2.15. Tables of the sub- and supergroups of the plane groups and space groups are contained in Parts 6 and 7. A general discussion of sub- and supergroups of crystallographic groups, together with further explanations and examples, is given in Section 8.3.3.

10.1.4. Noncrystallographic point groups

10.1.4.1. *Description of general point groups*

In Sections 10.1.2 and 10.1.3, only the 32 *crystallographic* point groups (crystal classes) are considered. In addition, infinitely many *noncrystallographic* point groups exist that are of interest as possible symmetries of molecules and of quasicrystals and as approximate local site symmetries in crystals. Crystallographic *and* noncrystallographic point groups are collected here under the name *general point groups.* They are reviewed in this section and listed in Tables 10.1.4.1 to 10.1.4.3.

Because of the infinite number of these groups only *classes of general point groups* (*general classes*)* can be listed. They are grouped into *general systems*, which are similar to the crystal systems. The 'general classes' are of two kinds: in the cubic, icosahedral, circular, cylindrical and spherical system, each general class contains *one* point group only, whereas in the $4N$-gonal, $(2N+1)$-gonal and $(4N+2)$-gonal system, each general class contains *infinitely* many point groups, which differ in their principal n-fold symmetry axis, with $n = 4, 8, 12, \ldots$ for the $4N$-gonal system, $n = 1, 3, 5, \ldots$ for the $(2N+1)$-gonal system and $n = 2, 6, 10, \ldots$ for the $(4N+2)$-gonal system.

Furthermore, some general point groups are of order infinity because they contain symmetry axes (rotation or rotoinversion axes) of order infinity† (∞-fold axes). These point groups occur in the

* The 'classes of general point groups' are not the same as the commonly used 'crystal classes' because some of them contain point groups of *different orders.* All these orders, however, follow a common scheme. In this sense, the 'general classes' are an extension of the concept of (geometric) crystal classes. For example, the general class *nmm* of the $4N$-gonal system contains the point groups $4mm$ (tetragonal), $8mm$ (octagonal), $12mm$ (dodecagonal), $16mm$ *etc.*

† The axes of order infinity, as considered here, do not correspond to cyclic groups (as do the axes of finite order) because there is no smallest rotation from which all other rotations can be derived as higher powers, *i.e.* by successive application. Instead, rotations of all possible angles exist. Nevertheless, it is customary to symbolize these axes as ∞ or C_∞; note that the Hermann–Mauguin symbols ∞/m and $\overline{\infty}$ are equivalent, and so are the Schoenflies symbols $C_{\infty h}, S_\infty$ and $C_{\infty i}$. (There exist also axes of order infinity that do correspond to cyclic groups, namely axes based upon smallest rotations with irrational values of the rotation angle.)

references