10.1. CRYSTALLOGRAPHIC AND NONCRYSTALLOGRAPHIC POINT GROUPS

Table 10.1.1.2. *The 32 three-dimensional crystallographic point groups, arranged according to crystal system (cf. Chapter 2.1)*

Full Hermann–Mauguin (left) and Schoenflies symbols (right). Dashed lines separate point groups with different Laue classes within one crystal system.

General symbol	Crystal system						
	Triclinic		Monoclinic (top) Orthorhombic (bottom)		Tetragonal		Trigonal
n	1	C_1	2	C_2	4	C_4	3
$\bar{n}$	$\bar{1}$	C_i	$m \equiv \bar{2}$	C_s	$\bar{4}$	S_4	$\bar{3}$
n/m			$2/m$	C_{2h}	$4/m$	C_{4h}	–
$n22$			222	D_2	422	D_4	32
nmm			$mm2$	C_{2v}	$4mm$	C_{4v}	$3m$
$\bar{n}2m$			–	–	$\bar{4}2m$	D_{2d}	$\bar{3}2/m$
$n/m\,2/m\,2/m$			$2/m\,2/m\,2/m$	D_{2h}	$4/m\,2/m\,2/m$	D_{4h}	–

Trigonal	Hexagonal		Cubic	
C_3	6	C_6	23	T
C_{3i}	$\bar{6} \equiv 3/m$	C_{3h}	–	–
–	$6/m$	C_{6h}	$2/m\bar{3}$	T_h
D_3	622	D_6	432	O
C_{3v}	$6mm$	C_{6v}	–	–
D_{3d}	$\bar{6}2m$	D_{3h}	$\bar{4}3m$	T_d
–	$6/m\,2/m\,2/m$	D_{6h}	$4/m\,\bar{3}\,2/m$	O_h

10.1.2. Crystallographic point groups

10.1.2.1. *Description of point groups*

In crystallography, point groups usually are described
(i) by means of their Hermann–Mauguin or Schoenflies symbols;
(ii) by means of their stereographic projections;
(iii) by means of the matrix representations of their symmetry operations, frequently listed in the form of Miller indices (*hkl*) of the equivalent general crystal faces;
(iv) by means of drawings of actual crystals, natural or synthetic. Descriptions (i) through (iii) are given in this section, whereas for crystal drawings and actual photographs reference is made to textbooks of crystallography and mineralogy; this also applies to the construction and the properties of the stereographic projection.

In Tables 10.1.2.1 and 10.1.2.2, the two- and three-dimensional crystallographic point groups are listed and described. The tables are arranged according to crystal systems and Laue classes. Within each crystal system and Laue class, the sequence of the point groups corresponds to that in the space-group tables of this volume: pure rotation groups are followed by groups containing reflections, rotoinversions and inversions. The holohedral point group is always given last.

In Tables 10.1.2.1 and 10.1.2.2, some point groups are described in *two or three versions*, in order to bring out the relations to the corresponding space groups (*cf.* Section 2.2.3):

(*a*) The three monoclinic point groups 2, *m* and $2/m$ are given with two settings, one with 'unique axis *b*' and one with 'unique axis *c*'.

(*b*) The two point groups $\bar{4}2m$ and $\bar{6}m2$ are described for two orientations with respect to the crystal axes, as $\bar{4}2m$ and $\bar{4}m2$ and as $\bar{6}m2$ and $\bar{6}2m$.

(*c*) The five trigonal point groups 3, $\bar{3}$, 32, $3m$ and $\bar{3}m$ are treated with two axial systems, 'hexagonal axes' and 'rhombohedral axes'.

(*d*) The hexagonal-axes description of the three trigonal point groups 32, $3m$ and $\bar{3}m$ is given for two orientations, as 321 and 312, as $3m1$ and $31m$, and as $\bar{3}m1$ and $\bar{3}1m$; this applies also to the two-dimensional point group $3m$.

The presentation of the point groups is similar to that of the space groups in Part 7. The *headline* contains the short Hermann–Mauguin and the Schoenflies symbols. The full Hermann–Mauguin symbol, if different, is given below the short symbol. No Schoenflies symbols exist for two-dimensional groups. For an explanation of the symbols see Section 2.2.4 and Chapter 12.1.

Next to the headline, a pair of *stereographic projections* is given. The diagram on the left displays a general crystal or point form, that on the right shows the 'framework of symmetry elements'. Except as noted below, the *c* axis is always normal to the plane of the figure,

the *a* axis points down the page and the *b* axis runs horizontally from left to right. For the five trigonal point groups, the *c* axis is normal to the page only for the description with 'hexagonal axes'; if described with 'rhombohedral axes', the direction [111] is normal and the positive *a* axis slopes towards the observer. The conventional coordinate systems used for the various crystal systems are listed in Table 2.1.2.1 and illustrated in Figs. 2.2.6.1 to 2.2.6.10.

In the *right-hand projection*, the graphical symbols of the symmetry elements are the same as those used in the space-group diagrams; they are listed in Chapter 1.4. Note that the symbol of a symmetry centre, a small circle, is also used for a face-pole in the left-hand diagram. Mirror planes are indicated by heavy solid lines or circles; thin lines are used for the projection circle, for symmetry axes in the plane and for some special zones in the cubic system.

In the *left-hand projection*, the projection circle and the coordinate axes are indicated by thin solid lines, as are again some special zones in the cubic system. The dots and circles in this projection can be interpreted in two ways.

(i) As *general face poles*, where they represent general crystal faces which form a polyhedron, the 'general crystal form' (face form) {*hkl*} of the point group (see below). In two dimensions, edges, edge poles, edge forms and polygons take the place of faces, face poles, crystal forms (face forms) and polyhedra in three dimensions.

Face poles marked as dots lie above the projection plane and represent faces which intersect the positive *c* axis* (positive Miller index *l*), those marked as circles lie below the projection plane (negative Miller index *l*). In two dimensions, edge poles always lie on the pole circle.

(ii) As *general points* (centres of atoms) that span a polyhedron or polygon, the 'general crystallographic point form' *x*, *y*, *z*. This interpretation is of interest in the study of coordination polyhedra, atomic groups and molecular shapes. The polyhedron or polygon of a point form is dual to the polyhedron of the corresponding crystal form.†

The general, special and limiting *crystal forms* and *point forms* constitute the main part of the table for each point group. The theoretical background is given below under *Crystal and point forms*; the explanation of the listed data is to be found under *Description of crystal and point forms*.

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* This does not apply to 'rhombohedral axes': here the positive directions of all three axes slope upwards from the plane of the paper: *cf.* Fig. 2.2.6.9.
† Dual polyhedra have the same number of edges, but the numbers of faces and vertices are interchanged; *cf.* textbooks of geometry.

references