

## 10. POINT GROUPS AND CRYSTAL CLASSES

Table 10.2.1.2. *Polar axes and nonpolar directions in the 21 noncentrosymmetric crystal classes*

All directions normal to an evenfold rotation axis and along rotoinversion axes are nonpolar. All directions other than those in the column ‘Nonpolar directions’ are polar. A symbol like  $[u0w]$  refers to the set of directions obtained for all possible values of  $u$  and  $w$ , in this case to all directions normal to the  $b$  axis, i.e. parallel to the plane (010). Symmetrically equivalent sets of nonpolar directions are placed between semicolons; the sequence of these sets follows the sequence of the symmetry directions in Table 2.2.4.1

System	Class	Polar (symmetry) axes	Nonpolar directions
Triclinic	1	None	None
Monoclinic Unique axis $b$	2	$[010]$	$[u0w]$
	$m$	None	$[010]$
Monoclinic Unique axis $c$	2	$[001]$	$[uv0]$
	$m$	None	$[001]$
Orthorhombic	222 $mm2$	None $[001]$	$[0vw]; [u0w]; [uv0]$ $[uv0]$
Tetragonal	4	$[001]$	$[uv0]$
	$\bar{4}$	None	$[001]; [uv0]$
	422	None	$[uv0]; [0vw] [u0w];$ $[uuw] [\bar{u}\bar{u}w]$
	$4mm$	$[001]$	$[uv0]$
	$\bar{4}2m$	None	$[uv0]; [0vw] [u0w]$
	$\bar{4}m2$	None	$[uv0]; [uuw] [\bar{u}\bar{u}w]$
Trigonal (Hexagonal axes)	3	$[001]$	None
	321	$[100], [010], [\bar{1}\bar{0}]$	$[u2uw] [\bar{2}\bar{u}\bar{u}w] [\bar{u}\bar{u}w]$
	312	$[\bar{1}\bar{0}], [120], [\bar{2}\bar{1}0]$	$[uuw] [\bar{u}0w] [0\bar{v}w]$
	3m1	$[001]$	$[100] [010] [\bar{1}\bar{0}]$
	31m	$[001]$	$[\bar{1}\bar{0}] [120] [\bar{2}\bar{1}0]$
Trigonal (Rhombohedral axes)	3	$[111]$	None
	32	$[\bar{1}\bar{0}], [01\bar{1}], [\bar{1}01]$	$[uuw] [uvv] [uvu]$
	3m	$[111]$	$[\bar{1}\bar{0}] [01\bar{1}] [\bar{1}01]$
Hexagonal	6	$[001]$	$[uv0]$
	$\bar{6}$	None	$[001]$
	622	None	$[u2uw] [\bar{2}\bar{u}\bar{u}w] [u\bar{u}w]$ $[uuw] [\bar{u}0w] [0\bar{v}w]$
	$6mm$	$[001]$	$[uv0]$
	$\bar{6}m2$	$[\bar{1}\bar{0}], [120], [\bar{2}\bar{1}0]$	$[uuw] [\bar{u}0w] [0\bar{v}w]$
	$\bar{6}2m$	$[100], [010], [\bar{1}\bar{0}]$	$[u2uw] [\bar{2}\bar{u}\bar{u}w] [u\bar{u}w]$
Cubic	23 $\bar{4}3m$	Four threefold axes along $\langle 111 \rangle$	$[0vw] [u0w] [uv0]$ $[0vw] [u0w] [uv0]$ $[0vw] [u0w] [uv0];$ $[uuw] [uvv] [uvu];$ $[u\bar{u}w] [\bar{u}\bar{v}\bar{v}] [\bar{u}\bar{v}u]$
	432	None	$\{$

The observation of etch figures is important when the morphological analysis is ambiguous (cf. Section 10.2.2). For instance, a tetragonal pyramid, which is compatible with point groups 4 and  $4mm$ , can be uniquely attributed to point group 4 if its face symmetry is found to be 1. For face symmetry  $m$ , group  $4mm$  would result. The (oriented) face symmetries of the 47 crystal forms in the various point groups are listed in column 6 of Table 10.1.2.3 and in column 3 of Table 10.1.2.2.

In noncentrosymmetric crystals, the etch pits on parallel but opposite faces, even though they have the same symmetry, may be

Table 10.2.4.1. *Categories of crystal systems distinguished according to the different forms of the indicatrix*

Crystal system	Shape of indicatrix	Optical character
Cubic	Sphere	Isotropic (not doubly refracting)
Tetragonal	Rotation ellipsoid	Uniaxial
Trigonal		Anisotropic (doubly refracting)
Hexagonal		
Orthorhombic		
Monoclinic	General ellipsoid	Biaxial
Triclinic		

of different size or shape, thus proving the absence of a symmetry centre. Note that the orientation of etch pits with respect to the edges of the face is significant (cf. Buerger, 1956), as well as the mutual arrangement of etch pits on opposite faces. Thus, for a pinacoid with face symmetry 1, the possible point groups  $\bar{1}$ , 2 and  $m$  of the crystal can be distinguished by the mutual orientation of etch pits on the two faces. Moreover, twinning by merohedry and the true symmetry of the two (or more) twin partners ('twin domains') may be detected.

The method of etching can be applied not only to growth faces but also to cleavage faces or arbitrarily cut faces.

## 10.2.4. Optical properties

Optical studies provide good facilities to determine the symmetry of transparent crystals. The following optical properties may be used.

## 10.2.4.1. Refraction

The dependence of the *refractive index* on the vibration direction of a plane-polarized light wave travelling through the crystal can be obtained from the optical indicatrix. This surface is an ellipsoid which can degenerate into a rotation ellipsoid or even into a sphere. Thus, the lowest symmetry of the property ‘refraction’ is  $2/m$   $2/m$   $2/m$ , the point group of the general ellipsoid. According to the three different forms of the indicatrix three categories of crystal systems have to be distinguished (Table 10.2.4.1).

The orientation of the indicatrix is related to the symmetry directions of the crystal. In tetragonal, trigonal and hexagonal crystals, the rotation axis of the indicatrix (which is the unique optic axis) is parallel to the main symmetry axis. For orthorhombic crystals, the three principal axes of the indicatrix are oriented parallel to the three symmetry directions. In the monoclinic system, one of the axes of the indicatrix coincides with the monoclinic symmetry direction, whereas, in the triclinic case, the indicatrix can, in principle, have any orientation relative to a chosen reference system. Thus, in triclinic and, with restrictions, in monoclinic crystals, the *orientation* of the indicatrix can change with wavelength  $\lambda$  and temperature  $T$  (orientation dispersion). In any system, the *size* of the indicatrix and, in all but the cubic system, its *shape* can also vary with  $\lambda$  and  $T$ .

When studying the symmetry of a crystal by optical means, note that strain can lower the apparent symmetry owing to the high sensitivity of optical properties to strain.

## 10.2.4.2. Optical activity

The symmetry information obtained from *optical activity* is quite different from that given by optical refraction. Optical activity is in principle confined to the 21 noncentrosymmetric classes but it can occur in only 15 of them (Table 10.2.1.1). In the 11 enantiomorph-