International Tables for Crystallography (2006). Vol. A, References, p. 844.

Table 13.2.2.1. *Three-dimensional derivative lattices of indices 2 to 7* (*cont.*)

Index 7	$1(7\mathbf{a},\mathbf{b},\mathbf{c})$; $2(\mathbf{a},7\mathbf{b},\mathbf{c})$; $3(\mathbf{a},\mathbf{b},7\mathbf{c})$; $4(7\mathbf{a},\mathbf{b}+\mathbf{a},\mathbf{c})$; $5(7\mathbf{a},\mathbf{b},\mathbf{c}+\mathbf{b})$; $6(\mathbf{a},7\mathbf{b},\mathbf{c}+\mathbf{b})$; $7(7\mathbf{a},\mathbf{b}-\mathbf{a},\mathbf{c})$; $8(7\mathbf{a},\mathbf{b},\mathbf{c}-\mathbf{a})$; $9(\mathbf{a},7\mathbf{b},\mathbf{c}-\mathbf{b})$; $10(7\mathbf{a},\mathbf{b}+2\mathbf{a},\mathbf{c})$; $11(7\mathbf{a},\mathbf{b},\mathbf{c}-3\mathbf{a})$; $12(\mathbf{a},7\mathbf{b},\mathbf{c}+2\mathbf{b})$; $13(7\mathbf{a},\mathbf{b}-3\mathbf{a},\mathbf{c})$; $14(7\mathbf{a},\mathbf{b},\mathbf{c}+2\mathbf{a})$; $15(\mathbf{a},7\mathbf{b},\mathbf{c}-3\mathbf{b})$; $16(7\mathbf{a},\mathbf{b}-2\mathbf{a},\mathbf{c})$; $17(7\mathbf{a},\mathbf{b},\mathbf{c}+3\mathbf{a})$; $18(\mathbf{a},7\mathbf{b},\mathbf{c}-2\mathbf{b})$; $19(7\mathbf{a},\mathbf{b}+3\mathbf{a},\mathbf{c})$; $20(7\mathbf{a},\mathbf{b},\mathbf{c}-2\mathbf{a})$; $21(\mathbf{a},7\mathbf{b},\mathbf{c}+3\mathbf{b})$; $22(7\mathbf{a},\mathbf{b}+\mathbf{a},\mathbf{c}+\mathbf{a})$; $23(7\mathbf{a},\mathbf{b}+\mathbf{a},\mathbf{c}-\mathbf{a})$; $24(7\mathbf{a},\mathbf{b}-\mathbf{a},\mathbf{c}+\mathbf{a})$; $25(7\mathbf{a},\mathbf{b}-\mathbf{a},\mathbf{c}-\mathbf{a})$; $26(7\mathbf{a},\mathbf{b}+\mathbf{a},\mathbf{c}+2\mathbf{a})$; $27(7\mathbf{a},\mathbf{b}+\mathbf{a},\mathbf{c}-2\mathbf{a})$; $28(7\mathbf{a},\mathbf{b}+2\mathbf{a},\mathbf{c}+\mathbf{a})$; $29(7\mathbf{a},\mathbf{b}-2\mathbf{a},\mathbf{c}+\mathbf{a})$; $30(7\mathbf{a},\mathbf{b}+3\mathbf{a},\mathbf{c}-3\mathbf{a})$; $31(7\mathbf{a},\mathbf{b}-3\mathbf{a},\mathbf{c}+3\mathbf{a})$; $32(7\mathbf{a},\mathbf{b}+\mathbf{a},\mathbf{c}+3\mathbf{a})$; $33(7\mathbf{a},\mathbf{b}+\mathbf{a},\mathbf{c}-3\mathbf{a})$; $34(7\mathbf{a},\mathbf{b}+3\mathbf{a},\mathbf{c}+\mathbf{a})$; $35(7\mathbf{a},\mathbf{b}-3\mathbf{a},\mathbf{c}+\mathbf{a})$; $36(7\mathbf{a},\mathbf{b}+2\mathbf{a},\mathbf{c}-2\mathbf{a})$; $37(7\mathbf{a},\mathbf{b}-2\mathbf{a},\mathbf{c}+2\mathbf{a})$; $38(7\mathbf{a},\mathbf{b}+2\mathbf{a},\mathbf{c}+2\mathbf{a})$; $39(7\mathbf{a},\mathbf{b}-3\mathbf{a},\mathbf{c}-\mathbf{a})$; $40(7\mathbf{a},\mathbf{b}-\mathbf{a},\mathbf{c}-3\mathbf{a})$; $41(7\mathbf{a},\mathbf{b}-2\mathbf{a},\mathbf{c}-2\mathbf{a})$; $42(7\mathbf{a},\mathbf{b}+3\mathbf{a},\mathbf{c}-\mathbf{a})$; $43(7\mathbf{a},\mathbf{b}-\mathbf{a},\mathbf{c}+3\mathbf{a})$; $44(7\mathbf{a},\mathbf{b}+2\mathbf{a},\mathbf{c}+3\mathbf{a})$; $45(7\mathbf{a},\mathbf{b}+3\mathbf{a},\mathbf{c}+2\mathbf{a})$; $46(7\mathbf{a},\mathbf{b}-2\mathbf{a},\mathbf{c}-3\mathbf{a})$; $47(7\mathbf{a},\mathbf{b}-3\mathbf{a},\mathbf{c}+2\mathbf{a})$; $48(7\mathbf{a},\mathbf{b}+2\mathbf{a},\mathbf{c}-3\mathbf{a})$; $49(7\mathbf{a},\mathbf{b}-3\mathbf{a},\mathbf{c}-2\mathbf{a})$; $50(7\mathbf{a},\mathbf{b}-2\mathbf{a},\mathbf{c}+3\mathbf{a})$; $51(7\mathbf{a},\mathbf{b}+3\mathbf{a},\mathbf{c}-2\mathbf{a})$; $52(7\mathbf{a},\mathbf{b}+3\mathbf{a},\mathbf{c}+3\mathbf{a})$; $53(7\mathbf{a},\mathbf{b}-2\mathbf{a},\mathbf{c}-\mathbf{a})$; $54(7\mathbf{a},\mathbf{b}-\mathbf{a},\mathbf{c}-2\mathbf{a})$; $55(7\mathbf{a},\mathbf{b}-3\mathbf{a},\mathbf{c}-3\mathbf{a})$; $56(7\mathbf{a},\mathbf{b}+2\mathbf{a},\mathbf{c}-\mathbf{a})$; $57(7\mathbf{a},\mathbf{b}-\mathbf{a},\mathbf{c}+2\mathbf{a})$

Example

One can derive easily the 7 derivative lattices of index 2.

$$1(2\mathbf{a},\mathbf{b},\mathbf{c});\ 2(\mathbf{a},2\mathbf{b},\mathbf{c});\ 3(\mathbf{a},\mathbf{b},2\mathbf{c});\ 4(2\mathbf{a},\mathbf{b}+\mathbf{a},\mathbf{c});$$
$$5(2\mathbf{a},\mathbf{b},\mathbf{c}+\mathbf{a});\ 6(\mathbf{a},2\mathbf{b},\mathbf{c}+\mathbf{b});\ 7(2\mathbf{a},\mathbf{b}+\mathbf{a},\mathbf{c}+\mathbf{a}).$$

Another primitive cell of a given derivative lattice is obtained if one of the following three elementary transformations is performed on the vectors of a primitive cell of this derivative lattice:

(i) the sign of a vector is changed;
(ii) two vectors are interchanged;
(iii) to a vector is added q times another vector (q integer).

(i) and (ii) are left-handed transformations, (iii) is right-handed.

Example

The primitive cell $\mathbf{a}'', \mathbf{b}'', \mathbf{c}''$ ($\mathbf{a}''=-2\mathbf{b}-\mathbf{a}$, $\mathbf{b}''=9\mathbf{a}-2\mathbf{c}-2\mathbf{b}$, $\mathbf{c}''=\mathbf{c}-\mathbf{a}+3\mathbf{b}$) belongs to the derivative lattice of index 10 given by the primitive cell $\mathbf{a}', \mathbf{b}', \mathbf{c}'$ ($\mathbf{a}'=5\mathbf{a}$, $\mathbf{b}'=2\mathbf{b}+\mathbf{a}$, $\mathbf{c}'=\mathbf{c}-2\mathbf{a}+\mathbf{b}$) because these two cells are related by the following sequence of elementary transformations:

$$(-2\mathbf{b}-\mathbf{a},\ 9\mathbf{a}-2\mathbf{c}-2\mathbf{b},\ \mathbf{c}-\mathbf{a}+3\mathbf{b})\overset{\text{(iii)}}{\rightarrow}$$
$$(-2\mathbf{b}-\mathbf{a},\ 9\mathbf{a}-2\mathbf{c}-2\mathbf{b},\ \mathbf{c}-2\mathbf{a}+\mathbf{b})\overset{\text{(iii)}}{\rightarrow}$$
$$(-2\mathbf{b}-\mathbf{a},\ 5\mathbf{a},\ \mathbf{c}-2\mathbf{a}+\mathbf{b})\overset{\text{(i)}}{\rightarrow}(2\mathbf{b}+\mathbf{a},\ 5\mathbf{a},\ \mathbf{c}-2\mathbf{a}+\mathbf{b})$$
$$\overset{\text{(ii)}}{\rightarrow}(5\mathbf{a},\ 2\mathbf{b}+\mathbf{a},\ \mathbf{c}-2\mathbf{a}+\mathbf{b});$$

($\mathbf{a}''$, $\mathbf{b}''$, $\mathbf{c}''$) and ($\mathbf{a}'$, $\mathbf{b}'$, $\mathbf{c}'$) have the same handedness.

Table 13.2.3.1. *Two-dimensional derivative lattices of indices 2 to 7*

The entry for each derivative lattice starts with a running number which is followed, between parentheses, by the appropriate basis-vector relations.

Index 2	$1(2\mathbf{a},\mathbf{b})$; $2(\mathbf{a},2\mathbf{b})$; $3(2\mathbf{a},\mathbf{b}+\mathbf{a})$
Index 3	$1(3\mathbf{a},\mathbf{b})$; $2(\mathbf{a},3\mathbf{b})$; $3(3\mathbf{a},\mathbf{b}+\mathbf{a})$; $4(3\mathbf{a},\mathbf{b}-\mathbf{a})$
Index 4	$1(4\mathbf{a},\mathbf{b})$; $2(\mathbf{a},4\mathbf{b})$; $3(4\mathbf{a},\mathbf{b}+\mathbf{a})$; $4(4\mathbf{a},\mathbf{b}-\mathbf{a})$; $5(4\mathbf{a},\mathbf{b}+2\mathbf{a})$; $6(2\mathbf{a},2\mathbf{b}+\mathbf{a})$; $7(2\mathbf{a},2\mathbf{b})$
Index 5	$1(5\mathbf{a},\mathbf{b})$; $2(\mathbf{a},5\mathbf{b})$; $3(5\mathbf{a},\mathbf{b}+\mathbf{a})$; $4(5\mathbf{a},\mathbf{b}-\mathbf{a})$; $5(5\mathbf{a},\mathbf{b}+2\mathbf{a})$; $6(5\mathbf{a},\mathbf{b}-2\mathbf{a})$
Index 6	$1(6\mathbf{a},\mathbf{b})$; $2(\mathbf{a},6\mathbf{b})$; $3(6\mathbf{a},\mathbf{b}+\mathbf{a})$; $4(6\mathbf{a},\mathbf{b}-\mathbf{a})$; $5(6\mathbf{a},\mathbf{b}+2\mathbf{a})$; $6(3\mathbf{a},2\mathbf{b}+\mathbf{a})$; $7(6\mathbf{a},\mathbf{b}-2\mathbf{a})$; $8(3\mathbf{a},2\mathbf{b}-\mathbf{a})$; $9(6\mathbf{a},\mathbf{b}+3\mathbf{a})$; $10(2\mathbf{a},3\mathbf{b}+\mathbf{a})$; $11(3\mathbf{a},2\mathbf{b})$; $12(2\mathbf{a},3\mathbf{b})$
Index 7	$1(7\mathbf{a},\mathbf{b})$; $2(\mathbf{a},7\mathbf{b})$; $3(7\mathbf{a},\mathbf{b}+\mathbf{a})$; $4(7\mathbf{a},\mathbf{b}-\mathbf{a})$; $5(7\mathbf{a},\mathbf{b}+2\mathbf{a})$; $6(7\mathbf{a},\mathbf{b}-3\mathbf{a})$; $7(7\mathbf{a},\mathbf{b}-2\mathbf{a})$; $8(7\mathbf{a},\mathbf{b}+3\mathbf{a})$

13.2.3. Two-dimensional derivative lattices

All previous considerations are valid also for two-dimensional lattices and their derivative lattices (Table 13.2.3.1). The relevant formula for any index is

$$\mathbf{a}'=p_1\mathbf{a},\quad \mathbf{b}'=p_2\mathbf{b}+q\mathbf{a}$$
[$\mathbf{a}$ direction is kept]
(p_1, p_2 positive integers, not necessarily prime;
index $=p_1p_2>1$; q any integer; $-p_1/2<q\le p_1/2$).

A similar formula is obtained by interchange of **a** and **b**.

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13.1

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13.2

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