# 15.4. Normalizers of point groups

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Normalizers with respect to the Euclidean or affine group may be defined for any group of isometries (*cf.* Gubler, 1982*a*,*b*). For a point group, however, it seems inadequate to use a supergroup that contains transformations that do not map a fixed point of that point group onto itself. Appropriate supergroups for the definition of normalizers of point groups are the full isometry groups of the sphere, $m\overline{\infty}$, and of the circle, $\infty m$, in three-dimensional and two-dimensional space (*cf.* Galiulin, 1978).

These normalizers are listed in Tables 15.4.1.1 and 15.4.1.2. It has to be noticed that the normalizer of a crystallographic point group may contain continuous rotations, *i.e.* rotations with infinitesimal rotation angle, or noncrystallographic rotations ($\infty m$; $m\overline{\infty}, \infty/mm, 8mm, 12mm$; $8/mmm, 12/mmm$). In analogy to space groups, these normalizers define equivalence relationships on the 'Wyckoff positions' of the point groups (*cf.* Section 10.1.2). They give also the relation between the different but equivalent morphological descriptions of a crystal.

Table 15.4.1.1. *Normalizers of the two-dimensional point groups with respect to the full isometry group of the circle*

The upper part refers to the crystallographic, the lower part to the noncrystallographic point groups as listed in Table 10.1.4.1.

| Normalizer | Point groups |
|---|---|
| $\infty m$ | 1, 2, 4, 3, 6 |
| $12mm$ | $6mm$ |
| $8mm$ | $4mm$ |
| $6mm$ | $3m$ |
| $4mm$ | $2mm$ |
| $2mm$ | $m$ |
| $\infty m$ | $n, \infty, \infty m$ |
| $(2n)mm$ | $nmm, nm$ |

Table 15.4.1.2. *Normalizers of the three-dimensional point groups with respect to the full isometry group of the sphere*

The upper part refers to the crystallographic, the lower part to the noncrystallographic point groups as listed in Table 10.1.4.2.

| Normalizer | Point groups |
|---|---|
| $m\overline{\infty}$ | $1, \overline{1}$ |
| $m\overline{3}m$ | $222, mmm, 23, m\overline{3}, 432, \overline{4}3m, m\overline{3}m$ |
| $\infty/mm$ | $2, m, 2/m, 4, \overline{4}, 4/m, 3, \overline{3}, 6, \overline{6}, 6/m$ |
| $12/mmm$ | $622, 6mm, 6/mmm$ |
| $8/mmm$ | $422, 4mm, 4/mmm$ |
| $6/mmm$ | $32, 3m, \overline{3}m, \overline{6}2m$ |
| $4/mmm$ | $mm2, \overline{4}2m$ |
| $m\overline{\infty}$ | $2\infty, m\overline{\infty}$ |
| $m\overline{35}$ | $235, m\overline{35}$ |
| $\infty/mm$ | $n, \overline{n}, n/m, \infty, \infty/m, \infty 2, \infty m, \infty/mm$ |
| $(2n)/mmm$ | $n22, nmm, n/mmm, n2, nm, \overline{n}m$ |
| $n/mmm$ | $\overline{n}2m$ |





references