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1.4. Graphical symbols for symmetry elements in one, two and three dimensions

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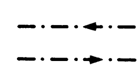
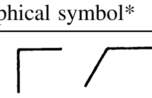
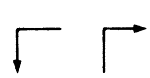
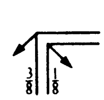
1.4.1. Symmetry planes normal to the plane of projection (three dimensions) and symmetry lines in the plane of the figure (two dimensions)

Symmetry plane or symmetry line	Graphical symbol	Glide vector in units of lattice translation vectors parallel and normal to the projection plane	Printed symbol
Reflection plane, mirror plane Reflection line, mirror line (two dimensions)		None	*m*
'Axial' glide plane Glide line (two dimensions)		$\frac{1}{2}$ lattice vector along line in projection plane $\frac{1}{2}$ lattice vector along line in figure plane	*a*, *b* or *c* *g*
'Axial' glide plane		$\frac{1}{2}$ lattice vector normal to projection plane	*a*, *b* or *c*
'Double' glide plane* (in centred cells only)		*Two* glide vectors: $\frac{1}{2}$ along line parallel to projection plane and $\frac{1}{2}$ normal to projection plane	*e*
'Diagonal' glide plane		*One* glide vector with *two* components: $\frac{1}{2}$ along line parallel to projection plane, $\frac{1}{2}$ normal to projection plane	*n*
'Diamond' glide plane† (pair of planes; in centred cells only)		$\frac{1}{4}$ along line parallel to projection plane, combined with $\frac{1}{4}$ normal to projection plane (arrow indicates direction parallel to the projection plane for which the normal component is positive)	*d*

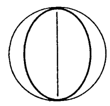
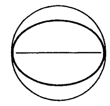
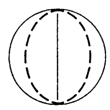
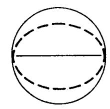
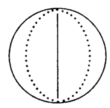
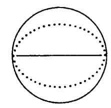
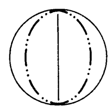
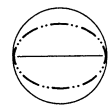
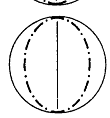
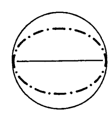
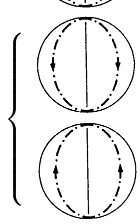
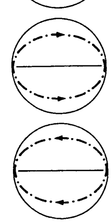
* For further explanations of the 'double' glide plane *e* see Note (iv) below and Note (x) in Section 1.3.2.
† See footnote § to Section 1.3.1.

1.4.2. Symmetry planes parallel to the plane of projection

Symmetry plane	Graphical symbol*	Glide vector in units of lattice translation vectors parallel to the projection plane	Printed symbol
Reflection plane, mirror plane		None	*m*
'Axial' glide plane		$\frac{1}{2}$ lattice vector in the direction of the arrow	*a*, *b* or *c*
'Double' glide plane† (in centred cells only)		*Two* glide vectors: $\frac{1}{2}$ in either of the directions of the two arrows	*e*
'Diagonal' glide plane		*One* glide vector with *two* components $\frac{1}{2}$ in the direction of the arrow	*n*
'Diamond' glide plane‡ (pair of planes; in centred cells only)		$\frac{1}{2}$ in the direction of the arrow; the glide vector is always half of a centring vector, *i.e.* one quarter of a diagonal of the conventional face-centred cell	*d*

* The symbols are given at the upper left corner of the space-group diagrams. A fraction h attached to a symbol indicates two symmetry planes with 'heights' h and $h + \frac{1}{2}$ above the plane of projection; *e.g.* $\frac{1}{8}$ stands for $h = \frac{1}{8}$ and $\frac{5}{8}$. No fraction means $h = 0$ and $\frac{1}{2}$ (*cf.* Section 2.2.6).
† For further explanations of the 'double' glide plane *e* see Note (iv) below and Note (x) in Section 1.3.2.
‡ See footnote § to Section 1.3.1.

1.4.3. Symmetry planes inclined to the plane of projection (in cubic space groups of classes $\bar{4}3m$ and $m\bar{3}m$ only)

Symmetry plane	Graphical symbol* for planes normal to		Glide vector in units of lattice translation vectors for planes normal to		Printed symbol
	[011] and [01$\bar{1}$]	[101] and [10$\bar{1}$]	[011] and [01$\bar{1}$]	[101] and [10$\bar{1}$]	
Reflection plane, mirror plane			None	None	*m*
‘Axial’ glide plane			$\frac{1}{2}$ lattice vector along [100]	$\frac{1}{2}$ lattice vector along [010]	*a* or *b*
‘Axial’ glide plane			$\frac{1}{2}$ lattice vector along [01$\bar{1}$] or along [011]	$\frac{1}{2}$ lattice vector along [10$\bar{1}$] or along [101]	
‘Double’ glide plane† [in space groups $I\bar{4}3m$ (217) and $Im\bar{3}m$ (229) only]			*Two* glide vectors: $\frac{1}{2}$ along [100] *and* $\frac{1}{2}$ along [01$\bar{1}$] or $\frac{1}{2}$ along [011]	*Two* glide vectors: $\frac{1}{2}$ along [010] *and* $\frac{1}{2}$ along [10$\bar{1}$] or $\frac{1}{2}$ along [101]	*e*
‘Diagonal’ glide plane			*One* glide vector: $\frac{1}{2}$ along [11$\bar{1}$] or along [111]‡	*One* glide vector: $\frac{1}{2}$ along [11$\bar{1}$] or along [111]‡	*n*
‘Diamond’ glide plane¶ (pair of planes; in centred cells only)			$\frac{1}{2}$ along [1$\bar{1}$1] or along [111]§	$\frac{1}{2}$ along [$\bar{1}$11] or along [111]§	*d*
			$\frac{1}{2}$ along [$\bar{1}\bar{1}$1] or along [$\bar{1}$11]§	$\frac{1}{2}$ along [$\bar{1}\bar{1}$1] or along [1$\bar{1}$1]§	

* The symbols represent orthographic projections. In the cubic space-group diagrams, complete orthographic projections of the symmetry elements around high-symmetry points, such as $0,0,0$; $\frac{1}{2},0,0$; $\frac{1}{4},\frac{1}{4},0$, are given as ‘inserts’.
† For further explanations of the ‘double’ glide plane *e* see Note (iv) below and Note (x) in Section 1.3.2.
‡ In the space groups $F\bar{4}3m$ (216), $Fm\bar{3}m$ (225) and $Fd\bar{3}m$ (227), the shortest lattice translation vectors in the glide directions are $\mathbf{t}(1,\frac{1}{2},\bar{\frac{1}{2}})$ or $\mathbf{t}(1,\frac{1}{2},\frac{1}{2})$ and $\mathbf{t}(\frac{1}{2},1,\bar{\frac{1}{2}})$ or $\mathbf{t}(\frac{1}{2},1,\frac{1}{2})$, respectively.
§ The glide vector is half of a centring vector, *i.e.* one quarter of the diagonal of the conventional body-centred cell in space groups $I\bar{4}3d$ (220) and $Ia\bar{3}d$ (230).
¶ See footnote § to Section 1.3.1.

1.4.4. Notes on graphical symbols of symmetry planes

(i) The *graphical* symbols and their explanations (columns 2 and 3) are independent of the projection direction and the labelling of the basis vectors. They are, therefore, applicable to any projection diagram of a space group. The *printed* symbols of *glide planes* (column 4), however, may change with a change of the basis vectors, as shown by the following example.

In the rhombohedral space groups $R3c$ (161) and $R\bar{3}c$ (167), the dotted line refers to a *c* glide when described with ‘hexagonal axes’ and projected along [001]; for a description with ‘rhombohedral axes’ and projection along [111], the same dotted glide plane would be called *n*. The dash-dotted *n* glide in the hexagonal description becomes an *a*, *b* or *c* glide in the rhombohedral description; *cf.* footnote † to Section 1.3.1.

(ii) The graphical symbols for glide planes in column 2 are not only used for the glide planes defined in Chapter 1.3, but also for the further glide planes *g* which are mentioned in Section 1.3.2 (Note x) and listed in Table 4.3.2.1; they are explained in Sections 2.2.9 and 11.1.2.

(iii) In monoclinic space groups, the ‘parallel’ glide vector of a glide plane may be along a lattice translation vector which is inclined to the projection plane.

(iv) In 1992, the International Union of Crystallography introduced the ‘double’ glide plane *e* and the graphical symbol ··–··– for *e* glide planes oriented ‘normal’ and ‘inclined’ to the plane of projection (de Wolff *et al.*, 1992); for details of *e* glide planes see Chapter 1.3. Note that the graphical symbol for *e* glide planes oriented ‘parallel’ to the projection plane has already been used in *IT* (1935) and *IT* (1952).

1.4.5. Symmetry axes normal to the plane of projection and symmetry points in the plane of the figure

Symmetry axis or symmetry point	Graphical symbol*	Screw vector of a right-handed screw rotation in units of the shortest lattice translation vector parallel to the axis	Printed symbol (partial elements in parentheses)
Identity	None	None	1
Twofold rotation axis Twofold rotation point (two dimensions)		None	2
Twofold screw axis: '2 sub 1'		$\frac{1}{2}$	2_1
Threefold rotation axis Threefold rotation point (two dimensions)		None	3
Threefold screw axis: '3 sub 1'		$\frac{1}{3}$	3_1
Threefold screw axis: '3 sub 2'		$\frac{2}{3}$	3_2
Fourfold rotation axis Fourfold rotation point (two dimensions)		None	4 (2)
Fourfold screw axis: '4 sub 1'		$\frac{1}{4}$	4_1 (2_1)
Fourfold screw axis: '4 sub 2'		$\frac{1}{2}$	4_2 (2)
Fourfold screw axis: '4 sub 3'		$\frac{3}{4}$	4_3 (2_1)
Sixfold rotation axis Sixfold rotation point (two dimensions)		None	6 (3,2)
Sixfold screw axis: '6 sub 1'		$\frac{1}{6}$	6_1 $(3_1, 2_1)$
Sixfold screw axis: '6 sub 2'		$\frac{1}{3}$	6_2 $(3_2, 2)$
Sixfold screw axis: '6 sub 3'		$\frac{1}{2}$	6_3 $(3, 2_1)$
Sixfold screw axis: '6 sub 4'		$\frac{2}{3}$	6_4 $(3_1, 2)$
Sixfold screw axis: '6 sub 5'		$\frac{5}{6}$	6_5 $(3_2, 2_1)$
Centre of symmetry, inversion centre: '1 bar' Reflection point, mirror point (one dimension)		None	$\bar{1}$
Inversion axis: '3 bar'		None	$\bar{3}$ $(3, \bar{1})$
Inversion axis: '4 bar'		None	$\bar{4}$ (2)
Inversion axis: '6 bar'		None	$\bar{6} \equiv 3/m$
Twofold rotation axis with centre of symmetry		None	$2/m$ $(\bar{1})$
Twofold screw axis with centre of symmetry		$\frac{1}{2}$	$2_1/m$ $(\bar{1})$
Fourfold rotation axis with centre of symmetry		None	$4/m$ $(\bar{4}, 2, \bar{1})$
'4 sub 2' screw axis with centre of symmetry		$\frac{1}{2}$	$4_2/m$ $(\bar{4}, 2, \bar{1})$
Sixfold rotation axis with centre of symmetry		None	$6/m$ $(\bar{6}, \bar{3}, 3, 2, \bar{1})$
'6 sub 3' screw axis with centre of symmetry		$\frac{1}{2}$	$6_3/m$ $(\bar{6}, \bar{3}, 3, 2_1, \bar{1})$

* Notes on the 'heights' h of symmetry points $\bar{1}$, $\bar{3}$, $\bar{4}$ and $\bar{6}$:

(1) Centres of symmetry $\bar{1}$ and $\bar{3}$, as well as inversion points $\bar{4}$ and $\bar{6}$ on $\bar{4}$ and $\bar{6}$ axes parallel to [001], occur in pairs at 'heights' h and $h + \frac{1}{2}$. In the space-group diagrams, only one fraction h is given, *e.g.* $\frac{1}{4}$ stands for $h = \frac{1}{4}$ and $\frac{3}{4}$. No fraction means $h = 0$ and $\frac{1}{2}$. In *cubic* space groups, however, because of their complexity, *both* fractions are given for vertical $\bar{4}$ axes, including $h = 0$ and $\frac{1}{2}$.

(2) Symmetries $4/m$ and $6/m$ contain vertical $\bar{4}$ and $\bar{6}$ axes; their $\bar{4}$ and $\bar{6}$ inversion points coincide with the centres of symmetry. This is not indicated in the space-group diagrams.

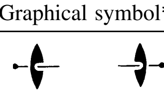
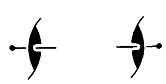
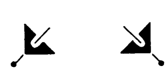
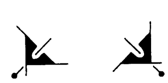
(3) Symmetries $4_2/m$ and $6_3/m$ also contain vertical $\bar{4}$ and $\bar{6}$ axes, but their $\bar{4}$ and $\bar{6}$ inversion points alternate with the centres of symmetry; *i.e.* $\bar{1}$ points at h and $h + \frac{1}{2}$ interleave with $\bar{4}$ or $\bar{6}$ points at $h + \frac{1}{4}$ and $h + \frac{3}{4}$. In the tetragonal and hexagonal space-group diagrams, only *one* fraction for $\bar{1}$ and one for $\bar{4}$ or $\bar{6}$ is given. In the cubic diagrams, *all four* fractions are listed for $4_2/m$; *e.g.* $Pm\bar{3}n$ (No. 223): $\bar{1}$: 0, $\frac{1}{2}$; $\bar{4}$: $\frac{1}{4}$, $\frac{3}{4}$.

1.4.6. Symmetry axes parallel to the plane of projection

Symmetry axis	Graphical symbol*	Screw vector of a right-handed screw rotation in units of the shortest lattice translation vector parallel to the axis	Printed symbol (partial elements in parentheses)
Twofold rotation axis		None	2
Twofold screw axis: '2 sub 1'		$\frac{1}{2}$	2_1
Fourfold rotation axis	 in cubic space groups only	None	4 (2)
Fourfold screw axis: '4 sub 1'		$\frac{1}{4}$	4_1 (2_1)
Fourfold screw axis: '4 sub 2'		$\frac{1}{2}$	4_2 (2)
Fourfold screw axis: '4 sub 3'		$\frac{3}{4}$	4_3 (2_1)
Inversion axis: '4 bar'		None	$\bar{4}$ (2)
Inversion point on '4 bar'-axis		–	$\bar{4}$ point

* The symbols for horizontal symmetry axes are given outside the unit cell of the space-group diagrams. *Twofold* axes always occur in pairs, at 'heights' h and $h+\frac{1}{2}$ above the plane of projection; here, a fraction h attached to such a symbol indicates two axes with heights h and $h+\frac{1}{2}$. No fraction stands for $h=0$ and $\frac{1}{2}$. The rule of pairwise occurrence, however, is not valid for the horizontal *fourfold* axes in cubic space groups; here, *all* heights are given, including $h=0$ and $\frac{1}{2}$. This applies also to the horizontal $\bar{4}$ axes and the $\bar{4}$ inversion points located on these axes.

1.4.7. Symmetry axes inclined to the plane of projection (in cubic space groups only)

Symmetry axis	Graphical symbol*	Screw vector of a right-handed screw rotation in units of the shortest lattice translation vector parallel to the axis	Printed symbol (partial elements in parentheses)
Twofold rotation axis	 Parallel to a face diagonal of the cube	None	2
Twofold screw axis: '2 sub 1'		$\frac{1}{2}$	2_1
Threefold rotation axis	 Parallel to a body diagonal of the cube	None	3
Threefold screw axis: '3 sub 1'		$\frac{1}{3}$	3_1
Threefold screw axis: '3 sub 2'		$\frac{2}{3}$	3_2
Inversion axis: '3 bar'		None	$\bar{3}$ $(3, \bar{1})$

* The dots mark the intersection points of axes with the plane at $h=0$. In some cases, the intersection points are obscured by symbols of symmetry elements with height $h \geq 0$; examples: $Fd\bar{3}$ (203), origin choice 2; $Pn\bar{3}n$ (222), origin choice 2; $Pm\bar{3}n$ (223); $Im\bar{3}m$ (229); $Ia\bar{3}d$ (230).

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1.2

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