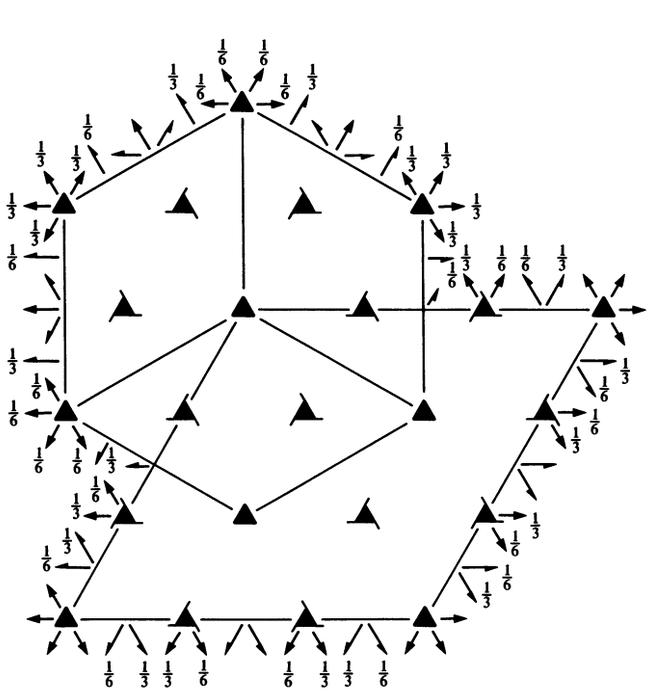
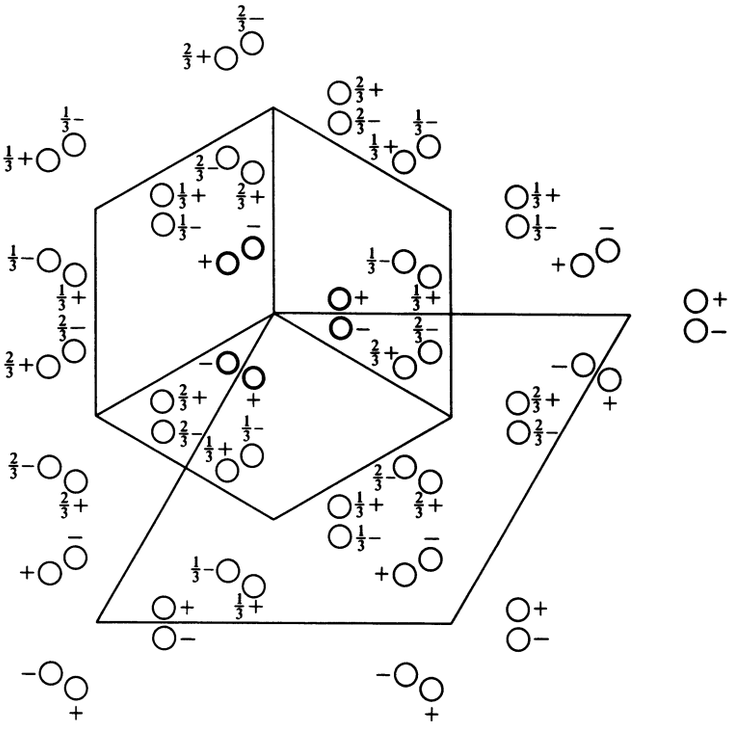
$R32$ D_3^7 32 Trigonal

No. 155 $R32$ Patterson symmetry $R\bar{3}m$

HEXAGONAL AXES





Origin at 32

Asymmetric unit $0 \le x \le \frac{2}{3}$; $0 \le y \le \frac{2}{3}$; $0 \le z \le \frac{1}{6}$; $x \le (1+y)/2$; $y \le \min(1-x,(1+x)/2)$

Vertices $0,0,0$ $\frac{1}{2},0,0$ $\frac{2}{3},\frac{1}{3},0$ $\frac{1}{3},\frac{2}{3},0$ $0,\frac{1}{2},0$
$0,0,\frac{1}{6}$ $\frac{1}{2},0,\frac{1}{6}$ $\frac{2}{3},\frac{1}{3},\frac{1}{6}$ $\frac{1}{3},\frac{2}{3},\frac{1}{6}$ $0,\frac{1}{2},\frac{1}{6}$

Symmetry operations

For $(0,0,0)+$ set

(1) 1 (2) 3^+ $0,0,z$ (3) 3^- $0,0,z$
(4) 2 $x,x,0$ (5) 2 $x,0,0$ (6) 2 $0,y,0$

For $(\frac{2}{3},\frac{1}{3},\frac{1}{3})+$ set

(1) $t(\frac{2}{3},\frac{1}{3},\frac{1}{3})$ (2) $3^+(0,0,\frac{1}{3})$ $\frac{1}{3},\frac{1}{3},z$ (3) $3^-(0,0,\frac{1}{3})$ $\frac{1}{3},0,z$
(4) $2(\frac{1}{2},\frac{1}{2},0)$ $x,x-\frac{1}{6},\frac{1}{6}$ (5) $2(\frac{1}{2},0,0)$ $x,\frac{1}{6},\frac{1}{6}$ (6) 2 $\frac{1}{3},y,\frac{1}{6}$

For $(\frac{1}{3},\frac{2}{3},\frac{2}{3})+$ set

(1) $t(\frac{1}{3},\frac{2}{3},\frac{2}{3})$ (2) $3^+(0,0,\frac{2}{3})$ $0,\frac{1}{3},z$ (3) $3^-(0,0,\frac{2}{3})$ $\frac{1}{3},\frac{1}{3},z$
(4) $2(\frac{1}{2},\frac{1}{2},0)$ $x,x+\frac{1}{6},\frac{1}{3}$ (5) 2 $x,\frac{1}{3},\frac{1}{3}$ (6) $2(0,\frac{1}{2},0)$ $\frac{1}{6},y,\frac{1}{3}$

Generators selected (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; $t(\frac{2}{3},\frac{1}{3},\frac{1}{3})$; (2); (4)

Positions

Multiplicity, Wyckoff letter, Site symmetry			Coordinates $(0,0,0)+$ $(\frac{2}{3},\frac{1}{3},\frac{1}{3})+$ $(\frac{1}{3},\frac{2}{3},\frac{2}{3})+$		
18	f	1	(1) x,y,z	(2) $\bar{y},x-y,z$	(3) $\bar{x}+y,\bar{x},z$
			(4) $y,x,\bar{z}$	(5) $x-y,\bar{y},\bar{z}$	(6) $\bar{x},\bar{x}+y,\bar{z}$
9	e	. 2	$x,0,\frac{1}{2}$	$0,x,\frac{1}{2}$	$\bar{x},\bar{x},\frac{1}{2}$
9	d	. 2	$x,0,0$	$0,x,0$	$\bar{x},\bar{x},0$
6	c	3 .	$0,0,z$	$0,0,\bar{z}$	
3	b	3 2	$0,0,\frac{1}{2}$		
3	a	3 2	$0,0,0$		

Reflection conditions

General:

$hkil$: $-h+k+l=3n$
$hki\underline{0}$: $-h+k=3n$
$hh\overline{2h}l$: $l=3n$
$h\bar{h}0l$: $h+l=3n$
$000l$: $l=3n$
$h\bar{h}00$: $h=3n$

Special: no extra conditions

Symmetry of special projections

Along [001] $p3m1$
$\mathbf{a}'=\frac{1}{3}(2\mathbf{a}+\mathbf{b})$ $\mathbf{b}'=\frac{1}{3}(-\mathbf{a}+\mathbf{b})$
Origin at $0,0,z$

Along [100] $p2$
$\mathbf{a}'=\frac{1}{2}(\mathbf{a}+2\mathbf{b})$ $\mathbf{b}'=\frac{1}{3}(-\mathbf{a}-2\mathbf{b}+\mathbf{c})$
Origin at $x,0,0$

Along [210] $p11m$
$\mathbf{a}'=\frac{1}{2}\mathbf{b}$ $\mathbf{b}'=\frac{1}{3}\mathbf{c}$
Origin at $x,\frac{1}{2}x,0$

Maximal non-isomorphic subgroups

I [2] $R31$ ($R3$, 146) (1; 2; 3)+
{ [3] $R12$ ($C2$, 5) (1; 4)+
[3] $R12$ ($C2$, 5) (1; 5)+
[3] $R12$ ($C2$, 5) (1; 6)+

IIa { [3] $P3_221$ (154) 1; 4; (2; 6) + $(\frac{1}{3},\frac{2}{3},\frac{2}{3})$; (3; 5) + $(\frac{2}{3},\frac{1}{3},\frac{1}{3})$
[3] $P3_221$ (154) 1; 5; (2; 4) + $(\frac{1}{3},\frac{2}{3},\frac{2}{3})$; (3; 6) + $(\frac{2}{3},\frac{1}{3},\frac{1}{3})$
[3] $P3_221$ (154) 1; 6; (2; 5) + $(\frac{1}{3},\frac{2}{3},\frac{2}{3})$; (3; 4) + $(\frac{2}{3},\frac{1}{3},\frac{1}{3})$
{ [3] $P3_121$ (152) 1; 4; (2; 6) + $(\frac{2}{3},\frac{1}{3},\frac{1}{3})$; (3; 5) + $(\frac{1}{3},\frac{2}{3},\frac{2}{3})$
[3] $P3_121$ (152) 1; 5; (2; 4) + $(\frac{2}{3},\frac{1}{3},\frac{1}{3})$; (3; 6) + $(\frac{1}{3},\frac{2}{3},\frac{2}{3})$
[3] $P3_121$ (152) 1; 6; (2; 5) + $(\frac{2}{3},\frac{1}{3},\frac{1}{3})$; (3; 4) + $(\frac{1}{3},\frac{2}{3},\frac{2}{3})$
{ [3] $P321$ (150) 1; 2; 3; 4; 5; 6
[3] $P321$ (150) 1; 2; 3; (4; 5; 6) + $(\frac{1}{3},\frac{2}{3},\frac{2}{3})$
[3] $P321$ (150) 1; 2; 3; (4; 5; 6) + $(\frac{2}{3},\frac{1}{3},\frac{1}{3})$

IIb none

Maximal isomorphic subgroups of lowest index

IIc [2] $R32$ ($\mathbf{a}'=-\mathbf{a},\mathbf{b}'=-\mathbf{b},\mathbf{c}'=2\mathbf{c}$) (155); [4] $R32$ ($\mathbf{a}'=-2\mathbf{a},\mathbf{b}'=-2\mathbf{b}$) (155)

Minimal non-isomorphic supergroups

I [2] $R\bar{3}m$ (166); [2] $R\bar{3}c$ (167); [4] $P432$ (207); [4] $P4_232$ (208); [4] $F432$ (209); [4] $F4_132$ (210); [4] $I432$ (211); [4] $P4_332$ (212); [4] $P4_132$ (213); [4] $I4_132$ (214)

II [3] $P312$ ($\mathbf{a}'=\frac{1}{3}(2\mathbf{a}+\mathbf{b}),\mathbf{b}'=\frac{1}{3}(-\mathbf{a}+\mathbf{b}),\mathbf{c}'=\frac{1}{3}\mathbf{c}$) (149)