

$R\bar{3}c$ 

No. 167

CONTINUED

## HEXAGONAL AXES

## Maximal non-isomorphic subgroups

<b>I</b>	[2] $R\bar{3}c$ (161)	(1; 2; 3; 10; 11; 12)+
	[2] $R\bar{3}2$ (155)	(1; 2; 3; 4; 5; 6)+
	[2] $R\bar{3}1$ ( $R\bar{3}$ , 148)	(1; 2; 3; 7; 8; 9)+
	{ [3] $R12/c$ ( $C2/c$ , 15)	(1; 4; 7; 10)+
	{ [3] $R12/c$ ( $C2/c$ , 15)	(1; 5; 7; 11)+
	{ [3] $R12/c$ ( $C2/c$ , 15)	(1; 6; 7; 12)+
<b>IIa</b>	{ [3] $P\bar{3}c1$ (165)	1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12
	{ [3] $P\bar{3}c1$ (165)	1; 2; 3; 10; 11; 12; (4; 5; 6; 7; 8; 9) + $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$
	{ [3] $P\bar{3}c1$ (165)	1; 2; 3; 10; 11; 12; (4; 5; 6; 7; 8; 9) + $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$
<b>IIb</b>	none	

## Maximal isomorphic subgroups of lowest index

**IIc** [4]  $R\bar{3}c$  ( $\mathbf{a}' = -2\mathbf{a}, \mathbf{b}' = -2\mathbf{b}$ ) (167); [5]  $R\bar{3}c$  ( $\mathbf{a}' = -\mathbf{a}, \mathbf{b}' = -\mathbf{b}, \mathbf{c}' = 5\mathbf{c}$ ) (167)

## Minimal non-isomorphic supergroups

- I** [4]  $Pn\bar{3}n$  (222); [4]  $Pm\bar{3}n$  (223); [4]  $Fm\bar{3}c$  (226); [4]  $Fd\bar{3}c$  (228); [4]  $Ia\bar{3}d$  (230)
- II** [2]  $R\bar{3}m$  ( $\mathbf{a}' = -\mathbf{a}, \mathbf{b}' = -\mathbf{b}, \mathbf{c}' = \frac{1}{2}\mathbf{c}$ ) (166); [3]  $P\bar{3}1c$  ( $\mathbf{a}' = \frac{1}{3}(2\mathbf{a} + \mathbf{b}), \mathbf{b}' = \frac{1}{3}(-\mathbf{a} + \mathbf{b}), \mathbf{c}' = \frac{1}{3}\mathbf{c}$ ) (163)

## RHOMBOHEDRAL AXES

## Maximal non-isomorphic subgroups

<b>I</b>	[2] $R\bar{3}c$ (161)	1; 2; 3; 10; 11; 12
	[2] $R\bar{3}2$ (155)	1; 2; 3; 4; 5; 6
	[2] $R\bar{3}1$ ( $R\bar{3}$ , 148)	1; 2; 3; 7; 8; 9
	{ [3] $R12/c$ ( $C2/c$ , 15)	1; 4; 7; 10
	{ [3] $R12/c$ ( $C2/c$ , 15)	1; 5; 7; 11
	{ [3] $R12/c$ ( $C2/c$ , 15)	1; 6; 7; 12
<b>IIa</b>	none	
<b>IIb</b>	[3] $P\bar{3}c1$ ( $\mathbf{a}' = \mathbf{a} - \mathbf{b}, \mathbf{b}' = \mathbf{b} - \mathbf{c}, \mathbf{c}' = \mathbf{a} + \mathbf{b} + \mathbf{c}$ ) (165)	

## Maximal isomorphic subgroups of lowest index

**IIc** [4]  $R\bar{3}c$  ( $\mathbf{a}' = -\mathbf{a} + \mathbf{b} + \mathbf{c}, \mathbf{b}' = \mathbf{a} - \mathbf{b} + \mathbf{c}, \mathbf{c}' = \mathbf{a} + \mathbf{b} - \mathbf{c}$ ) (167); [5]  $R\bar{3}c$  ( $\mathbf{a}' = \mathbf{a} + 2\mathbf{b} + 2\mathbf{c}, \mathbf{b}' = 2\mathbf{a} + \mathbf{b} + 2\mathbf{c}, \mathbf{c}' = 2\mathbf{a} + 2\mathbf{b} + \mathbf{c}$ ) (167)

## Minimal non-isomorphic supergroups

- I** [4]  $Pn\bar{3}n$  (222); [4]  $Pm\bar{3}n$  (223); [4]  $Fm\bar{3}c$  (226); [4]  $Fd\bar{3}c$  (228); [4]  $Ia\bar{3}d$  (230)
- II** [2]  $R\bar{3}m$  ( $\mathbf{a}' = \frac{1}{2}(-\mathbf{a} + \mathbf{b} + \mathbf{c}), \mathbf{b}' = \frac{1}{2}(\mathbf{a} - \mathbf{b} + \mathbf{c}), \mathbf{c}' = \frac{1}{2}(\mathbf{a} + \mathbf{b} - \mathbf{c})$ ) (166);  
 [3]  $P\bar{3}1c$  ( $\mathbf{a}' = \frac{1}{3}(2\mathbf{a} - \mathbf{b} - \mathbf{c}), \mathbf{b}' = \frac{1}{3}(-\mathbf{a} + 2\mathbf{b} - \mathbf{c}), \mathbf{c}' = \frac{1}{3}(\mathbf{a} + \mathbf{b} + \mathbf{c})$ ) (163)

Trigonal

$\bar{3}m$

$D_{3d}^6$

$R\bar{3}c$

Patterson symmetry  $R\bar{3}m$

$R\bar{3}2/c$

No. 167

RHOMBOHEDRAL AXES  
(For drawings see hexagonal axes)

Origin at centre ( $\bar{3}$ ) at  $\bar{3}c$

**Asymmetric unit**  $\frac{1}{4} \leq x \leq \frac{5}{4}; \frac{1}{4} \leq y \leq \frac{5}{4}; \frac{1}{4} \leq z \leq \frac{3}{4}; y \leq x; z \leq \min(y, \frac{3}{2} - x)$   
**Vertices**  $\frac{1}{4}, \frac{1}{4}, \frac{1}{4} \quad \frac{5}{4}, \frac{1}{4}, \frac{1}{4} \quad \frac{5}{4}, \frac{5}{4}, \frac{1}{4} \quad \frac{3}{4}, \frac{3}{4}, \frac{3}{4}$

### Symmetry operations

- |   |   |   |
|---|---|---|
| (1) 1   | (2) $3^+ x, x, x$                                       | (3) $3^- x, x, x$                                       |
| (4) $2 \bar{x} + \frac{1}{2}, \frac{1}{4}, x$           | (5) $2 x, \bar{x} + \frac{1}{2}, \frac{1}{4}$           | (6) $2 \frac{1}{4}, y + \frac{1}{2}, \bar{y}$           |
| (7) $\bar{1} 0, 0, 0$                                   | (8) $\bar{3}^+ x, x, x; 0, 0, 0$                        | (9) $\bar{3}^- x, x, x; 0, 0, 0$                        |
| (10) $n(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) x, y, x$ | (11) $n(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) x, x, z$ | (12) $n(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) x, y, y$ |

**Generators selected** (1);  $t(1, 0, 0)$ ;  $t(0, 1, 0)$ ;  $t(0, 0, 1)$ ; (2); (4); (7)

### Positions

Multiplicity,  
Wyckoff letter,  
Site symmetry

Coordinates

Reflection conditions

- |    |     |   |   |   |   |
|----|-----|---|---|---|---|
| 12 | $f$ | 1 | (1) $x, y, z$   | (2) $z, x, y$   | (3) $y, z, x$   |
|    |     |   | (4) $\bar{z} + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{x} + \frac{1}{2}$ | (5) $\bar{y} + \frac{1}{2}, \bar{x} + \frac{1}{2}, \bar{z} + \frac{1}{2}$ | (6) $\bar{x} + \frac{1}{2}, \bar{z} + \frac{1}{2}, \bar{y} + \frac{1}{2}$ |
|    |     |   | (7) $\bar{x}, \bar{y}, \bar{z}$   | (8) $\bar{z}, \bar{x}, \bar{y}$   | (9) $\bar{y}, \bar{z}, \bar{x}$   |
|    |     |   | (10) $z + \frac{1}{2}, y + \frac{1}{2}, x + \frac{1}{2}$                  | (11) $y + \frac{1}{2}, x + \frac{1}{2}, z + \frac{1}{2}$                  | (12) $x + \frac{1}{2}, z + \frac{1}{2}, y + \frac{1}{2}$                  |

General:

$hhl : l = 2n$   
 $hhh : h = 2n$

Special: as above, plus

no extra conditions

- |   |     |             |  |  |  |
|---|-----|-------------|--|--|--|
| 6 | $e$ | .2          | $x, \bar{x} + \frac{1}{2}, \frac{1}{4}$<br>$\bar{x}, x + \frac{1}{2}, \frac{3}{4}$ | $\frac{1}{4}, x, \bar{x} + \frac{1}{2}$<br>$\frac{3}{4}, \bar{x}, x + \frac{1}{2}$ | $\bar{x} + \frac{1}{2}, \frac{1}{4}, x$<br>$x + \frac{1}{2}, \frac{3}{4}, \bar{x}$ |
| 6 | $d$ | $\bar{1}$   | $\frac{1}{2}, 0, 0$  | $0, \frac{1}{2}, 0$  | $0, 0, \frac{1}{2}$  |
|   |     |             | $\frac{1}{2}, \frac{1}{2}, 0$  | $\frac{1}{2}, 0, \frac{1}{2}$  | $0, \frac{1}{2}, \frac{1}{2}$  |
| 4 | $c$ | 3.          | $x, x, x$  | $\bar{x} + \frac{1}{2}, \bar{x} + \frac{1}{2}, \bar{x} + \frac{1}{2}$              | $\bar{x}, \bar{x}, \bar{x}$  |
|   |     |             | $x + \frac{1}{2}, x + \frac{1}{2}, x + \frac{1}{2}$                                |  |  |
| 2 | $b$ | $\bar{3}$ . | $0, 0, 0$  | $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$  |  |
| 2 | $a$ | 32          | $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$  | $\frac{3}{4}, \frac{3}{4}, \frac{3}{4}$  |  |

$hkl : h + k + l = 2n$

$hkl : h + k + l = 2n$

$hkl : h + k + l = 2n$

$hkl : h + k + l = 2n$

### Symmetry of special projections

Along  $[111] p6mm$

$\mathbf{a}' = \frac{1}{3}(2\mathbf{a} - \mathbf{b} - \mathbf{c})$   $\mathbf{b}' = \frac{1}{3}(-\mathbf{a} + 2\mathbf{b} - \mathbf{c})$

Origin at  $x, x, x$

(Continued on preceding page)

Along  $[1\bar{1}0] p2$

$\mathbf{a}' = \frac{1}{2}(\mathbf{a} + \mathbf{b} - 2\mathbf{c})$   $\mathbf{b}' = \frac{1}{2}\mathbf{c}$

Origin at  $x, \bar{x}, 0$

Along  $[2\bar{1}\bar{1}] p2gm$

$\mathbf{a}' = \frac{1}{2}(\mathbf{b} - \mathbf{c})$   $\mathbf{b}' = \frac{1}{3}(\mathbf{a} + \mathbf{b} + \mathbf{c})$

Origin at  $2x, \bar{x}, \bar{x}$