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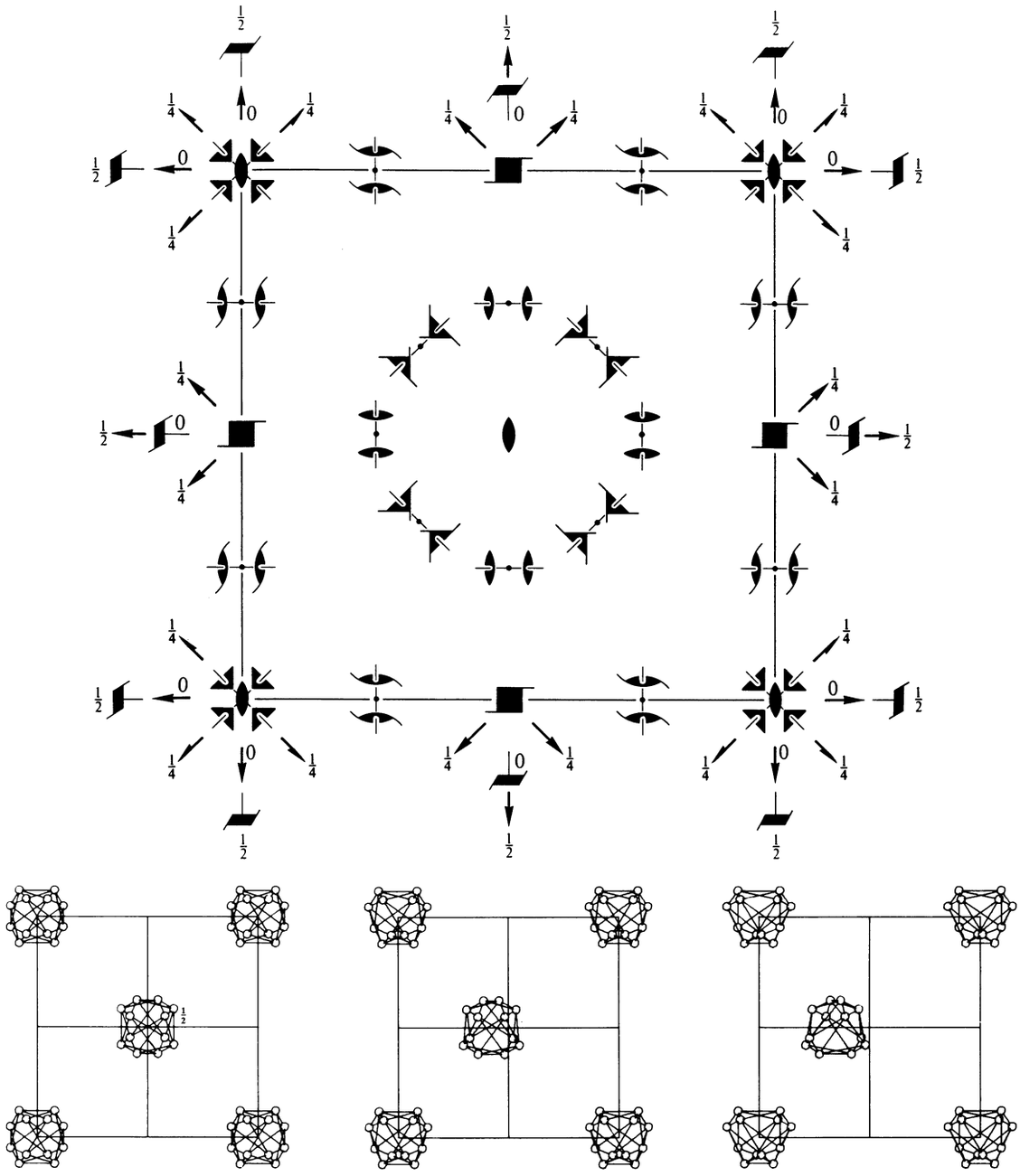
# $P4_232$ $O^2$ 432 Cubic

## No. 208 $P4_232$ Patterson symmetry $Pm\bar{3}m$



**Origin** at 23

**Asymmetric unit** $0 \le x \le \frac{1}{2}$; $0 \le y \le \frac{1}{2}$; $-\frac{1}{4} \le z \le \frac{1}{4}$; $\max(-x, x-\frac{1}{2}, -y, y-\frac{1}{2}) \le z \le \min(x, \frac{1}{2}-x, y, \frac{1}{2}-y)$

Vertices $0,0,0$ $\frac{1}{2},0,0$ $\frac{1}{2},\frac{1}{2},0$ $0,\frac{1}{2},0$ $\frac{1}{4},\frac{1}{4},\frac{1}{4}$ $\frac{1}{4},\frac{1}{4},-\frac{1}{4}$

**Symmetry operations**

| | | | |
|---|---|---|---|
| (1) $1$ | (2) $2 \quad 0,0,z$ | (3) $2 \quad 0,y,0$ | (4) $2 \quad x,0,0$ |
| (5) $3^+ \quad x,x,x$ | (6) $3^+ \quad \bar{x},x,\bar{x}$ | (7) $3^+ \quad x,\bar{x},\bar{x}$ | (8) $3^+ \quad \bar{x},\bar{x},x$ |
| (9) $3^- \quad x,x,x$ | (10) $3^- \quad x,\bar{x},\bar{x}$ | (11) $3^- \quad \bar{x},\bar{x},x$ | (12) $3^- \quad \bar{x},x,\bar{x}$ |
| (13) $2(\frac{1}{2},\frac{1}{2},0) \quad x,x,\frac{1}{4}$ | (14) $2 \quad x,\bar{x}+\frac{1}{2},\frac{1}{4}$ | (15) $4^-(0,0,\frac{1}{2}) \quad \frac{1}{2},0,z$ | (16) $4^+(0,0,\frac{1}{2}) \quad 0,\frac{1}{2},z$ |
| (17) $4^-(\frac{1}{2},0,0) \quad x,\frac{1}{2},0$ | (18) $2(0,\frac{1}{2},\frac{1}{2}) \quad \frac{1}{4},y,y$ | (19) $2 \quad \frac{1}{4},y+\frac{1}{2},\bar{y}$ | (20) $4^+(\frac{1}{2},0,0) \quad x,0,\frac{1}{2}$ |
| (21) $4^+(0,\frac{1}{2},0) \quad \frac{1}{2},y,0$ | (22) $2(\frac{1}{2},0,\frac{1}{2}) \quad x,\frac{1}{4},x$ | (23) $4^-(0,\frac{1}{2},0) \quad 0,y,\frac{1}{2}$ | (24) $2 \quad \bar{x}+\frac{1}{2},\frac{1}{4},x$ |

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**Generators selected** (1); $t(1,0,0)$; $t(0,1,0)$; $t(0,0,1)$; (2); (3); (5); (13)

**Positions**

| Multiplicity, Wyckoff letter, Site symmetry | | | Coordinates | | | | | | Reflection conditions |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | $h,k,l$ permutable<br>General: |
| 24 | $m$ | 1 | (1) $x,y,z$<br>(5) $z,x,y$<br>(9) $y,z,x$<br>(13) $y+\frac{1}{2},x+\frac{1}{2},\bar{z}+\frac{1}{2}$<br>(17) $x+\frac{1}{2},z+\frac{1}{2},\bar{y}+\frac{1}{2}$<br>(21) $z+\frac{1}{2},y+\frac{1}{2},\bar{x}+\frac{1}{2}$ | (2) $\bar{x},\bar{y},z$<br>(6) $z,\bar{x},\bar{y}$<br>(10) $\bar{y},z,\bar{x}$<br>(14) $\bar{y}+\frac{1}{2},\bar{x}+\frac{1}{2},\bar{z}+\frac{1}{2}$<br>(18) $\bar{x}+\frac{1}{2},z+\frac{1}{2},y+\frac{1}{2}$<br>(22) $z+\frac{1}{2},\bar{y}+\frac{1}{2},x+\frac{1}{2}$ | (3) $\bar{x},y,\bar{z}$<br>(7) $\bar{z},\bar{x},y$<br>(11) $y,\bar{z},\bar{x}$<br>(15) $y+\frac{1}{2},\bar{x}+\frac{1}{2},z+\frac{1}{2}$<br>(19) $\bar{x}+\frac{1}{2},\bar{z}+\frac{1}{2},\bar{y}+\frac{1}{2}$<br>(23) $\bar{z}+\frac{1}{2},y+\frac{1}{2},x+\frac{1}{2}$ | (4) $x,\bar{y},\bar{z}$<br>(8) $\bar{z},x,\bar{y}$<br>(12) $\bar{y},\bar{z},x$<br>(16) $\bar{y}+\frac{1}{2},x+\frac{1}{2},z+\frac{1}{2}$<br>(20) $x+\frac{1}{2},\bar{z}+\frac{1}{2},y+\frac{1}{2}$<br>(24) $\bar{z}+\frac{1}{2},\bar{y}+\frac{1}{2},\bar{x}+\frac{1}{2}$ | | | | $h00$ : $h=2n$ |
| | | | | | | | | | Special: as above, plus |
| 12 | $l$ | $..2$ | $\frac{1}{4},y,y+\frac{1}{2}$<br>$y+\frac{1}{2},\frac{1}{4},y$<br>$y,y+\frac{1}{2},\frac{1}{4}$ | $\frac{3}{4},\bar{y},y+\frac{1}{2}$<br>$y+\frac{1}{2},\frac{3}{4},\bar{y}$<br>$\bar{y},y+\frac{1}{2},\frac{3}{4}$ | $\frac{3}{4},y,\bar{y}+\frac{1}{2}$<br>$\bar{y}+\frac{1}{2},\frac{3}{4},y$<br>$y,\bar{y}+\frac{1}{2},\frac{3}{4}$ | $\frac{1}{4},\bar{y},\bar{y}+\frac{1}{2}$<br>$\bar{y}+\frac{1}{2},\frac{1}{4},\bar{y}$<br>$\bar{y},\bar{y}+\frac{1}{2},\frac{1}{4}$ | | | no extra conditions |
| 12 | $k$ | $..2$ | $\frac{1}{4},y,\bar{y}+\frac{1}{2}$<br>$\bar{y}+\frac{1}{2},\frac{1}{4},y$<br>$y,\bar{y}+\frac{1}{2},\frac{1}{4}$ | $\frac{3}{4},\bar{y},\bar{y}+\frac{1}{2}$<br>$\bar{y}+\frac{1}{2},\frac{3}{4},\bar{y}$<br>$\bar{y},\bar{y}+\frac{1}{2},\frac{3}{4}$ | $\frac{3}{4},y,y+\frac{1}{2}$<br>$y+\frac{1}{2},\frac{3}{4},y$<br>$y,y+\frac{1}{2},\frac{3}{4}$ | $\frac{1}{4},\bar{y},y+\frac{1}{2}$<br>$y+\frac{1}{2},\frac{1}{4},\bar{y}$<br>$\bar{y},y+\frac{1}{2},\frac{1}{4}$ | | | no extra conditions |
| 12 | $j$ | $2..$ | $x,\frac{1}{2},0$<br>$0,x+\frac{1}{2},\frac{1}{2}$ | $\bar{x},\frac{1}{2},0$<br>$0,\bar{x}+\frac{1}{2},\frac{1}{2}$ | $0,x,\frac{1}{2}$<br>$x+\frac{1}{2},\frac{1}{2},0$ | $0,\bar{x},\frac{1}{2}$<br>$\bar{x}+\frac{1}{2},\frac{1}{2},0$ | $\frac{1}{2},0,x$<br>$\frac{1}{2},0,\bar{x}+\frac{1}{2}$ | $\frac{1}{2},0,\bar{x}$<br>$\frac{1}{2},0,x+\frac{1}{2}$ | $hkl$ : $h=2n$<br>$hhl$ : $l=2n$ |
| 12 | $i$ | $2..$ | $x,0,\frac{1}{2}$<br>$\frac{1}{2},x+\frac{1}{2},0$ | $\bar{x},0,\frac{1}{2}$<br>$\frac{1}{2},\bar{x}+\frac{1}{2},0$ | $\frac{1}{2},x,0$<br>$x+\frac{1}{2},0,\frac{1}{2}$ | $\frac{1}{2},\bar{x},0$<br>$\bar{x}+\frac{1}{2},0,\frac{1}{2}$ | $0,\frac{1}{2},x$<br>$0,\frac{1}{2},\bar{x}+\frac{1}{2}$ | $0,\frac{1}{2},\bar{x}$<br>$0,\frac{1}{2},x+\frac{1}{2}$ | $hkl$ : $h=2n$<br>$hhl$ : $l=2n$ |
| 12 | $h$ | $2..$ | $x,0,0$<br>$\frac{1}{2},x+\frac{1}{2},\frac{1}{2}$ | $\bar{x},0,0$<br>$\frac{1}{2},\bar{x}+\frac{1}{2},\frac{1}{2}$ | $0,x,0$<br>$x+\frac{1}{2},\frac{1}{2},\frac{1}{2}$ | $0,\bar{x},0$<br>$\bar{x}+\frac{1}{2},\frac{1}{2},\frac{1}{2}$ | $0,0,x$<br>$\frac{1}{2},\frac{1}{2},\bar{x}+\frac{1}{2}$ | $0,0,\bar{x}$<br>$\frac{1}{2},\frac{1}{2},x+\frac{1}{2}$ | $hkl$ : $h+k+l=2n$ |
| 8 | $g$ | $.3.$ | $x,x,x$<br>$\bar{x},x,\bar{x}$<br>$x+\frac{1}{2},x+\frac{1}{2},\bar{x}+\frac{1}{2}$<br>$x+\frac{1}{2},\bar{x}+\frac{1}{2},x+\frac{1}{2}$ | $\bar{x},\bar{x},x$<br>$x,\bar{x},\bar{x}$<br>$\bar{x}+\frac{1}{2},\bar{x}+\frac{1}{2},\bar{x}+\frac{1}{2}$<br>$\bar{x}+\frac{1}{2},x+\frac{1}{2},x+\frac{1}{2}$ | | | | | $0kl$ : $k+l=2n$ |
| 6 | $f$ | $2.22$ | $\frac{1}{4},\frac{1}{2},0$ | $\frac{3}{4},\frac{1}{2},0$ | $0,\frac{1}{4},\frac{1}{2}$ | $0,\frac{3}{4},\frac{1}{2}$ | $\frac{1}{2},0,\frac{1}{4}$ | $\frac{1}{2},0,\frac{3}{4}$ | $hkl$ : $h+k+l=2n$<br>or $h=2n+1, k=4n$ and $l=4n+2$ |
| 6 | $e$ | $2.22$ | $\frac{1}{4},0,\frac{1}{2}$ | $\frac{3}{4},0,\frac{1}{2}$ | $\frac{1}{2},\frac{1}{4},0$ | $\frac{1}{2},\frac{3}{4},0$ | $0,\frac{1}{2},\frac{1}{4}$ | $0,\frac{1}{2},\frac{3}{4}$ | |
| 6 | $d$ | $222..$ | $0,\frac{1}{2},\frac{1}{2}$ | $\frac{1}{2},0,\frac{1}{2}$ | $\frac{1}{2},\frac{1}{2},0$ | $0,\frac{1}{2},0$ | $\frac{1}{2},0,0$ | $0,0,\frac{1}{2}$ | $hkl$ : $h+k+l=2n$ |
| 4 | $c$ | $.32$ | $\frac{3}{4},\frac{3}{4},\frac{3}{4}$ | $\frac{1}{4},\frac{1}{4},\frac{3}{4}$ | $\frac{1}{4},\frac{3}{4},\frac{1}{4}$ | $\frac{3}{4},\frac{1}{4},\frac{1}{4}$ | | | $hkl$ : $h+k,h+l,k+l=2n$ |
| 4 | $b$ | $.32$ | $\frac{1}{4},\frac{1}{4},\frac{1}{4}$ | $\frac{3}{4},\frac{3}{4},\frac{1}{4}$ | $\frac{3}{4},\frac{1}{4},\frac{3}{4}$ | $\frac{1}{4},\frac{3}{4},\frac{3}{4}$ | | | $hkl$ : $h+k,h+l,k+l=2n$ |
| 2 | $a$ | $23.$ | $0,0,0$ | $\frac{1}{2},\frac{1}{2},\frac{1}{2}$ | | | | | $hkl$ : $h+k+l=2n$ |

**Symmetry of special projections**

Along [001] $p4mm$
$\mathbf{a}'=\mathbf{a}$ $\mathbf{b}'=\mathbf{b}$
Origin at $0,\frac{1}{2},z$

Along [111] $p3m1$
$\mathbf{a}'=\frac{1}{3}(2\mathbf{a}-\mathbf{b}-\mathbf{c})$ $\mathbf{b}'=\frac{1}{3}(-\mathbf{a}+2\mathbf{b}-\mathbf{c})$
Origin at $x,x,x$

Along [110] $p2mm$
$\mathbf{a}'=\frac{1}{2}(-\mathbf{a}+\mathbf{b})$ $\mathbf{b}'=\mathbf{c}$
Origin at $x,x,\frac{1}{4}$

**Maximal non-isomorphic subgroups**

**I** [2] $P231$ ($P23$, 195) 1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12

[3] $P4_212$ ($P4_222$, 93) 1; 2; 3; 4; 13; 14; 15; 16
[3] $P4_212$ ($P4_222$, 93) 1; 2; 3; 4; 17; 18; 19; 20
[3] $P4_212$ ($P4_222$, 93) 1; 2; 3; 4; 21; 22; 23; 24

[4] $P132$ ($R32$, 155) 1; 5; 9; 14; 19; 24
[4] $P132$ ($R32$, 155) 1; 6; 12; 13; 18; 24
[4] $P132$ ($R32$, 155) 1; 7; 10; 13; 19; 22
[4] $P132$ ($R32$, 155) 1; 8; 11; 14; 18; 22

**IIa** none

**IIb** [2] $F4_132$ ($\mathbf{a}' = 2\mathbf{a}, \mathbf{b}' = 2\mathbf{b}, \mathbf{c}' = 2\mathbf{c}$) (210); [4] $I4_132$ ($\mathbf{a}' = 2\mathbf{a}, \mathbf{b}' = 2\mathbf{b}, \mathbf{c}' = 2\mathbf{c}$) (214)

**Maximal isomorphic subgroups of lowest index**

**IIc** [27] $P4_232$ ($\mathbf{a}' = 3\mathbf{a}, \mathbf{b}' = 3\mathbf{b}, \mathbf{c}' = 3\mathbf{c}$) (208)

**Minimal non-isomorphic supergroups**

**I** [2] $Pm\bar{3}n$ (223); [2] $Pn\bar{3}m$ (224)

**II** [2] $I432$ (211); [4] $F432$ (209)