

*I*432

$O^5$

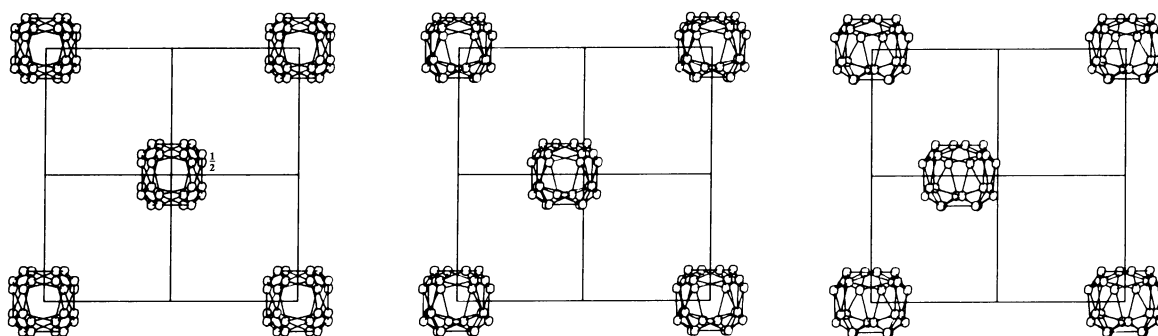
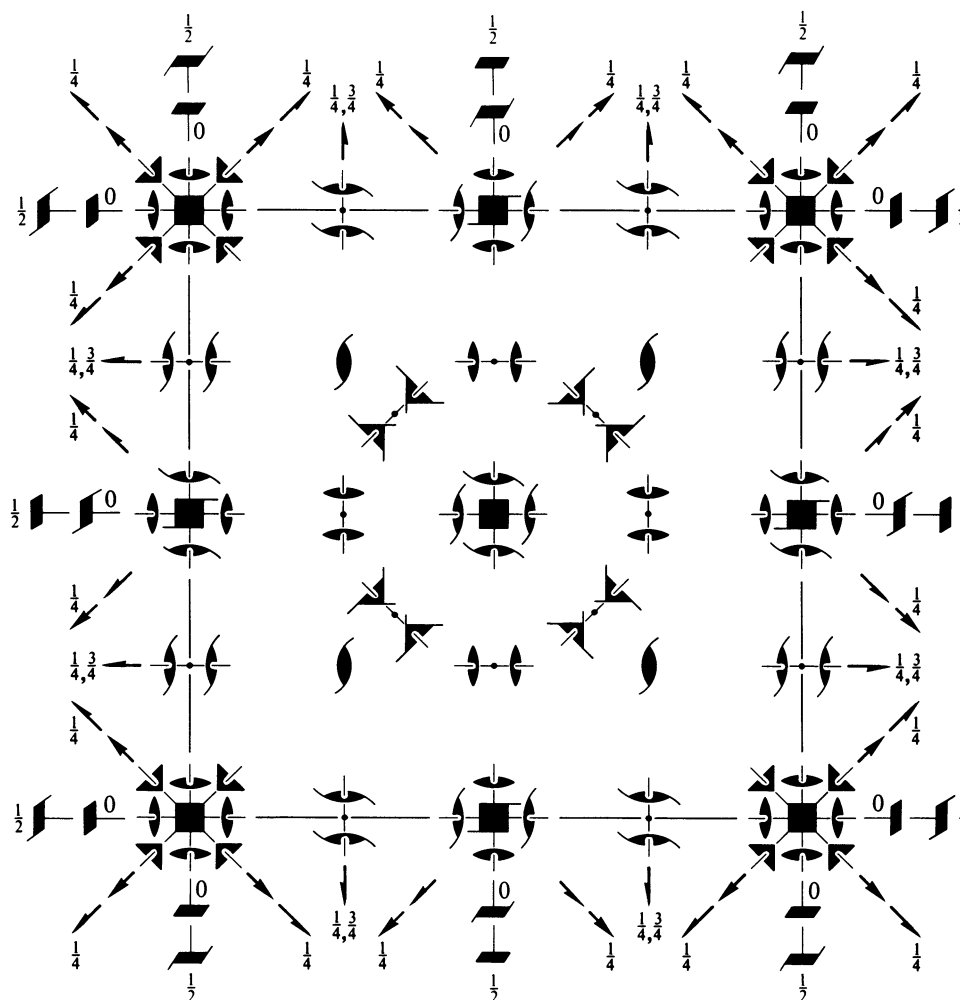
432

Cubic

No. 211

*I*432

Patterson symmetry  $Im\bar{3}m$



Origin at 432

Asymmetric unit  $0 \leq x \leq \frac{1}{2}; 0 \leq y \leq \frac{1}{2}; 0 \leq z \leq \frac{1}{4}; z \leq \min(x, \frac{1}{2} - x, y, \frac{1}{2} - y)$

Vertices  $0, 0, 0 \quad \frac{1}{2}, 0, 0 \quad \frac{1}{2}, \frac{1}{2}, 0 \quad 0, \frac{1}{2}, 0 \quad \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$

**Symmetry operations**

For (0,0,0)+ set

- |                           |  |  |   |
|---------------------------|--|--|---|
| (1) 1                     | (2) 2 0,0,z                                  | (3) 2 0,y,0                                  | (4) 2 x,0,0                                 |
| (5) 3 <sup>+</sup> x,x,x  | (6) 3 <sup>+</sup> $\bar{x}$ ,x, $\bar{x}$   | (7) 3 <sup>+</sup> x, $\bar{x}$ , $\bar{x}$  | (8) 3 <sup>+</sup> $\bar{x}$ , $\bar{x}$ ,x |
| (9) 3 <sup>-</sup> x,x,x  | (10) 3 <sup>-</sup> x, $\bar{x}$ , $\bar{x}$ | (11) 3 <sup>-</sup> $\bar{x}$ , $\bar{x}$ ,x | (12) 3 <sup>-</sup> $\bar{x}$ ,x, $\bar{x}$ |
| (13) 2 x,x,0              | (14) 2 x, $\bar{x}$ ,0                       | (15) 4 <sup>-</sup> 0,0,z                    | (16) 4 <sup>+</sup> 0,0,z                   |
| (17) 4 <sup>-</sup> x,0,0 | (18) 2 0,y,y                                 | (19) 2 0,y, $\bar{y}$                        | (20) 4 <sup>+</sup> x,0,0                   |
| (21) 4 <sup>+</sup> 0,y,0 | (22) 2 x,0,x                                 | (23) 4 <sup>-</sup> 0,y,0                    | (24) 2 $\bar{x}$ ,0,x                       |

For ( $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$ )+ set

- |  |  |  |  |
|--|--|--|--|
| (1) $t(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$                       | (2) 2(0,0, $\frac{1}{2}$ ) $\frac{1}{4}, \frac{1}{4}, z$   | (3) 2(0, $\frac{1}{2}$ ,0) $\frac{1}{4}, y, \frac{1}{4}$   | (4) 2( $\frac{1}{2}$ ,0,0) $x, \frac{1}{4}, \frac{1}{4}$   |
| (5) 3 <sup>+</sup> ( $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$ ) x,x,x | (6) 3 <sup>+</sup> ( $\frac{1}{6}, -\frac{1}{6}, \frac{1}{6}$ ) $\bar{x} + \frac{1}{3}, x + \frac{1}{3}, \bar{x}$  | (7) 3 <sup>+</sup> ( $-\frac{1}{6}, \frac{1}{6}, \frac{1}{6}$ ) $x + \frac{2}{3}, \bar{x} - \frac{1}{3}, \bar{x}$  | (8) 3 <sup>+</sup> ( $\frac{1}{6}, \frac{1}{6}, -\frac{1}{6}$ ) $\bar{x} + \frac{1}{3}, \bar{x} + \frac{2}{3}, x$  |
| (9) 3 <sup>-</sup> ( $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$ ) x,x,x | (10) 3 <sup>-</sup> ( $-\frac{1}{6}, \frac{1}{6}, \frac{1}{6}$ ) $x + \frac{1}{3}, \bar{x} + \frac{1}{3}, \bar{x}$ | (11) 3 <sup>-</sup> ( $\frac{1}{6}, \frac{1}{6}, -\frac{1}{6}$ ) $\bar{x} + \frac{2}{3}, \bar{x} + \frac{1}{3}, x$ | (12) 3 <sup>-</sup> ( $\frac{1}{6}, -\frac{1}{6}, \frac{1}{6}$ ) $\bar{x} - \frac{1}{3}, x + \frac{2}{3}, \bar{x}$ |
| (13) 2( $\frac{1}{2}, \frac{1}{2}, 0$ ) x,x, $\frac{1}{4}$           | (14) 2 x, $\bar{x} + \frac{1}{2}, \frac{1}{4}$   | (15) 4 <sup>-</sup> (0,0, $\frac{1}{2}$ ) $\frac{1}{2}, 0, z$  | (16) 4 <sup>+</sup> (0,0, $\frac{1}{2}$ ) 0, $\frac{1}{2}, z$  |
| (17) 4 <sup>-</sup> ( $\frac{1}{2}, 0, 0$ ) x, $\frac{1}{2}, 0$      | (18) 2(0, $\frac{1}{2}, \frac{1}{2}$ ) $\frac{1}{4}, y, y$   | (19) 2 $\frac{1}{4}, y + \frac{1}{2}, \bar{y}$   | (20) 4 <sup>+</sup> ( $\frac{1}{2}, 0, 0$ ) x,0, $\frac{1}{2}$   |
| (21) 4 <sup>+</sup> (0, $\frac{1}{2}, 0$ ) $\frac{1}{2}, y, 0$       | (22) 2( $\frac{1}{2}, 0, \frac{1}{2}$ ) x, $\frac{1}{4}, x$  | (23) 4 <sup>-</sup> (0, $\frac{1}{2}, 0$ ) 0,y, $\frac{1}{2}$  | (24) 2 $\bar{x} + \frac{1}{2}, \frac{1}{4}, x$   |

**Generators selected** (1);  $t(1,0,0)$ ;  $t(0,1,0)$ ;  $t(0,0,1)$ ;  $t(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ ; (2); (3); (5); (13)

**Positions**

Multiplicity, Wyckoff letter, Site symmetry	Coordinates				Reflection conditions
	(0,0,0)+ ( $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$ )+				
48 j 1	(1) x,y,z (5) z,x,y (9) y,z,x (13) y,x, $\bar{z}$ (17) x,z, $\bar{y}$ (21) z,y, $\bar{x}$	(2) $\bar{x}, \bar{y}, z$ (6) z, $\bar{x}, \bar{y}$ (10) $\bar{y}, z, \bar{x}$ (14) $\bar{y}, \bar{x}, \bar{z}$ (18) $\bar{x}, z, y$ (22) z, $\bar{y}, x$	(3) $\bar{x}, y, \bar{z}$ (7) $\bar{z}, \bar{x}, y$ (11) y, $\bar{z}, \bar{x}$ (15) y, $\bar{x}, z$ (19) $\bar{x}, \bar{z}, \bar{y}$ (23) $\bar{z}, y, x$	(4) x, $\bar{y}, \bar{z}$ (8) $\bar{z}, x, \bar{y}$ (12) $\bar{y}, \bar{z}, x$ (16) $\bar{y}, x, z$ (20) x, $\bar{z}, y$ (24) $\bar{z}, \bar{y}, \bar{x}$	hkl : h + k + l = 2n OkL : k + l = 2n hhl : l = 2n h00 : h = 2n
24 i .. 2	$\frac{1}{4}, y, \bar{y} + \frac{1}{2}$ $\bar{y} + \frac{1}{2}, \frac{1}{4}, y$ y, $\bar{y} + \frac{1}{2}, \frac{1}{4}$	$\frac{3}{4}, \bar{y}, \bar{y} + \frac{1}{2}$ $\bar{y} + \frac{1}{2}, \frac{3}{4}, \bar{y}$ $\bar{y}, \bar{y} + \frac{1}{2}, \frac{3}{4}$	$\frac{3}{4}, y, y + \frac{1}{2}$ $y + \frac{1}{2}, \frac{3}{4}, y$ y, $y + \frac{1}{2}, \frac{3}{4}$	$\frac{1}{4}, \bar{y}, y + \frac{1}{2}$ $y + \frac{1}{2}, \frac{1}{4}, \bar{y}$ $\bar{y}, y + \frac{1}{2}, \frac{1}{4}$	no extra conditions
24 h .. 2	0,y,y $\bar{y}, 0, y$	0, $\bar{y}, y$ $\bar{y}, 0, \bar{y}$	0,y, $\bar{y}$ y,y,0 $\bar{y}, y, 0$	0, $\bar{y}, \bar{y}$ y,0,y $\bar{y}, \bar{y}, 0$	no extra conditions
24 g 2..	x, $\frac{1}{2}, 0$ $\frac{1}{2}, x, 0$	$\bar{x}, \frac{1}{2}, 0$ $\frac{1}{2}, \bar{x}, 0$	0,x, $\frac{1}{2}$ x,0, $\frac{1}{2}$	0, $\bar{x}, \frac{1}{2}$ $\bar{x}, 0, \frac{1}{2}$ $\frac{1}{2}, 0, x$ 0, $\frac{1}{2}, \bar{x}$ $\frac{1}{2}, 0, \bar{x}$ 0, $\frac{1}{2}, x$	no extra conditions
16 f .3.	x,x,x x,x, $\bar{x}$	$\bar{x}, \bar{x}, x$ $\bar{x}, \bar{x}, \bar{x}$	$\bar{x}, x, \bar{x}$ x, $\bar{x}, x$	x, $\bar{x}, \bar{x}$ $\bar{x}, x, x$	no extra conditions
12 e 4..	x,0,0	$\bar{x}, 0, 0$	0,x,0	0, $\bar{x}, 0$ 0,0,x 0,0, $\bar{x}$	no extra conditions
12 d 2.22	$\frac{1}{4}, \frac{1}{2}, 0$	$\frac{3}{4}, \frac{1}{2}, 0$	0, $\frac{1}{4}, \frac{1}{2}$	0, $\frac{3}{4}, \frac{1}{2}$ $\frac{1}{2}, 0, \frac{1}{4}$ $\frac{1}{2}, 0, \frac{3}{4}$	no extra conditions
8 c .32	$\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$	$\frac{3}{4}, \frac{3}{4}, \frac{1}{4}$	$\frac{3}{4}, \frac{1}{4}, \frac{3}{4}$	$\frac{1}{4}, \frac{3}{4}, \frac{3}{4}$	hkl : k, l = 2n
6 b 42.2	0, $\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, 0, \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}, 0$		no extra conditions
2 a 432	0,0,0				no extra conditions

**Symmetry of special projections**

Along [001]  $p4mm$

$\mathbf{a}' = \frac{1}{2}(\mathbf{a} - \mathbf{b})$      $\mathbf{b}' = \frac{1}{2}(\mathbf{a} + \mathbf{b})$

Origin at 0,0,z

Along [111]  $p3m1$

$\mathbf{a}' = \frac{1}{3}(2\mathbf{a} - \mathbf{b} - \mathbf{c})$      $\mathbf{b}' = \frac{1}{3}(-\mathbf{a} + 2\mathbf{b} - \mathbf{c})$

Origin at x,x,x

Along [110]  $p2mm$

$\mathbf{a}' = \frac{1}{2}(-\mathbf{a} + \mathbf{b})$      $\mathbf{b}' = \frac{1}{2}\mathbf{c}$

Origin at x,x,0

(Continued on page 645)

**Maximal non-isomorphic subgroups**

- I**    [2]  $I231 (I23, 197)$     (1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12)+  
       { [3]  $I412 (I422, 97)$     (1; 2; 3; 4; 13; 14; 15; 16)+  
       { [3]  $I412 (I422, 97)$     (1; 2; 3; 4; 17; 18; 19; 20)+  
       { [3]  $I412 (I422, 97)$     (1; 2; 3; 4; 21; 22; 23; 24)+  
       { [4]  $I132 (R32, 155)$     (1; 5; 9; 14; 19; 24)+  
       { [4]  $I132 (R32, 155)$     (1; 6; 12; 13; 18; 24)+  
       { [4]  $I132 (R32, 155)$     (1; 7; 10; 13; 19; 22)+  
       { [4]  $I132 (R32, 155)$     (1; 8; 11; 14; 18; 22)+
- IIa**   [2]  $P4_232 (208)$         1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; (13; 14; 15; 16; 17; 18; 19; 20; 21; 22; 23; 24) +  $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$   
       [2]  $P432 (207)$         1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; 13; 14; 15; 16; 17; 18; 19; 20; 21; 22; 23; 24
- IIb**   none

**Maximal isomorphic subgroups of lowest index**

- IIc**   [27]  $I432 (a' = 3a, b' = 3b, c' = 3c) (211)$

**Minimal non-isomorphic supergroups**

- I**    [2]  $Im\bar{3}m (229)$
- II**   [4]  $P432 (a' = \frac{1}{2}a, b' = \frac{1}{2}b, c' = \frac{1}{2}c) (207)$