

$Fd\bar{3}m$

$O_h^7$

$m\bar{3}m$

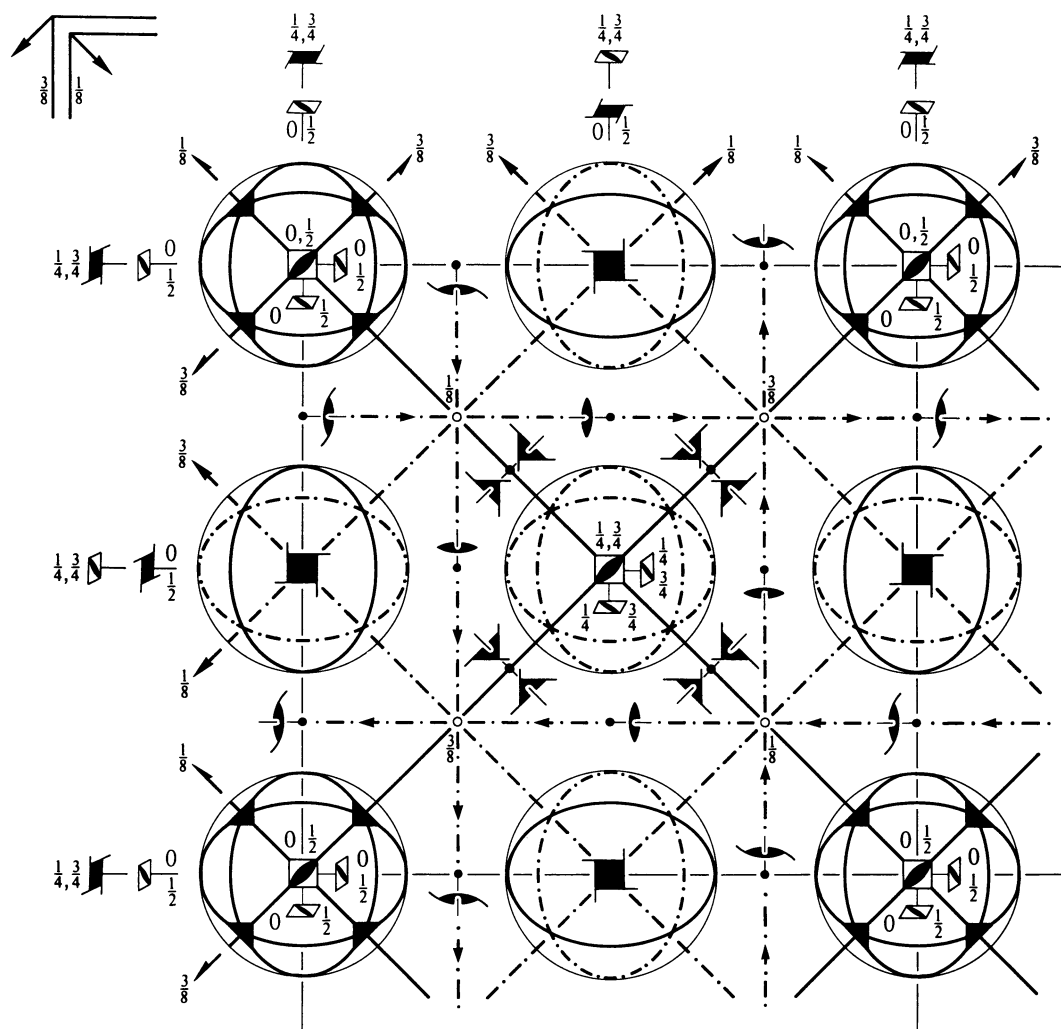
Cubic

No. 227

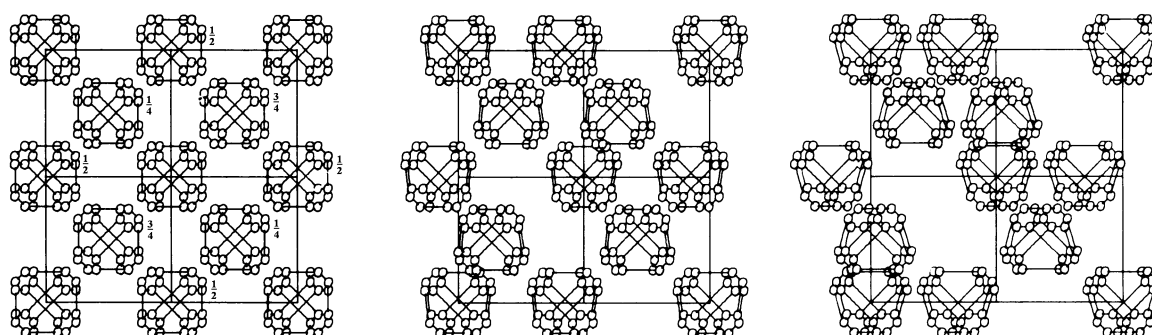
$F 4_1/d \bar{3} 2/m$

Patterson symmetry  $Fm\bar{3}m$

ORIGIN CHOICE 1



Upper left quadrant only



Origin at  $\bar{4}3m$ , at  $-\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}$  from centre ( $\bar{3}m$ )

Asymmetric unit  $0 \leq x \leq \frac{1}{2}; 0 \leq y \leq \frac{1}{8}; -\frac{1}{8} \leq z \leq \frac{1}{8}; y \leq \min(\frac{1}{2}-x, x); -y \leq z \leq y$

Vertices  $0, 0, 0 \quad \frac{1}{2}, 0, 0 \quad \frac{3}{8}, \frac{1}{8}, \frac{1}{8} \quad \frac{1}{8}, \frac{1}{8}, \frac{1}{8} \quad \frac{3}{8}, \frac{1}{8}, -\frac{1}{8} \quad \frac{1}{8}, \frac{1}{8}, -\frac{1}{8}$

Symmetry operations

(given on page 699)

**Generators selected** (1);  $t(1,0,0)$ ;  $t(0,1,0)$ ;  $t(0,0,1)$ ;  $t(0, \frac{1}{2}, \frac{1}{2})$ ;  $t(\frac{1}{2}, 0, \frac{1}{2})$ ; (2); (3); (5); (13); (25)

**Positions**

Multiplicity, Wyckoff letter, Site symmetry	Coordinates				Reflection conditions
	$(0,0,0)+$	$(0, \frac{1}{2}, \frac{1}{2})+$	$(\frac{1}{2}, 0, \frac{1}{2})+$	$(\frac{1}{2}, \frac{1}{2}, 0)+$	$h, k, l$ permutable
					General:

192	$i$	1	(1) $x, y, z$	(2) $\bar{x}, \bar{y} + \frac{1}{2}, z + \frac{1}{2}$	(3) $\bar{x} + \frac{1}{2}, y + \frac{1}{2}, \bar{z}$	(4) $x + \frac{1}{2}, \bar{y}, \bar{z} + \frac{1}{2}$	$hkl$ : $h + k = 2n$ and $h + l, k + l = 2n$
			(5) $z, x, y$	(6) $z + \frac{1}{2}, \bar{x}, \bar{y} + \frac{1}{2}$	(7) $\bar{z}, \bar{x} + \frac{1}{2}, y + \frac{1}{2}$	(8) $\bar{z} + \frac{1}{2}, x + \frac{1}{2}, \bar{y}$	$Ok$ : $k + l = 4n$ and $k, l = 2n$
			(9) $y, z, x$	(10) $\bar{y} + \frac{1}{2}, z + \frac{1}{2}, \bar{x}$	(11) $y + \frac{1}{2}, \bar{z}, \bar{x} + \frac{1}{2}$	(12) $\bar{y}, \bar{z} + \frac{1}{2}, x + \frac{1}{2}$	$hhl$ : $h + l = 2n$
			(13) $y + \frac{3}{4}, x + \frac{1}{4}, \bar{z} + \frac{3}{4}$	(14) $\bar{y} + \frac{1}{4}, \bar{x} + \frac{1}{4}, \bar{z} + \frac{1}{4}$	(15) $y + \frac{1}{4}, \bar{x} + \frac{3}{4}, z + \frac{3}{4}$	(16) $\bar{y} + \frac{3}{4}, x + \frac{3}{4}, z + \frac{1}{4}$	$h00$ : $h = 4n$
			(17) $x + \frac{3}{4}, z + \frac{1}{4}, \bar{y} + \frac{3}{4}$	(18) $\bar{x} + \frac{3}{4}, z + \frac{3}{4}, y + \frac{1}{4}$	(19) $\bar{x} + \frac{1}{4}, \bar{z} + \frac{1}{4}, \bar{y} + \frac{1}{4}$	(20) $x + \frac{1}{4}, \bar{z} + \frac{3}{4}, y + \frac{3}{4}$	
			(21) $z + \frac{3}{4}, y + \frac{1}{4}, \bar{x} + \frac{3}{4}$	(22) $z + \frac{1}{4}, \bar{y} + \frac{3}{4}, x + \frac{3}{4}$	(23) $\bar{z} + \frac{3}{4}, y + \frac{3}{4}, x + \frac{1}{4}$	(24) $\bar{z} + \frac{1}{4}, \bar{y} + \frac{1}{4}, \bar{x} + \frac{1}{4}$	
			(25) $\bar{x} + \frac{1}{4}, \bar{y} + \frac{1}{4}, \bar{z} + \frac{1}{4}$	(26) $x + \frac{1}{4}, y + \frac{3}{4}, \bar{z} + \frac{3}{4}$	(27) $x + \frac{3}{4}, \bar{y} + \frac{3}{4}, z + \frac{1}{4}$	(28) $\bar{x} + \frac{3}{4}, y + \frac{1}{4}, z + \frac{3}{4}$	
			(29) $\bar{z} + \frac{1}{4}, \bar{x} + \frac{1}{4}, \bar{y} + \frac{1}{4}$	(30) $\bar{z} + \frac{3}{4}, x + \frac{1}{4}, y + \frac{3}{4}$	(31) $z + \frac{1}{4}, x + \frac{3}{4}, \bar{y} + \frac{3}{4}$	(32) $z + \frac{3}{4}, \bar{x} + \frac{3}{4}, y + \frac{1}{4}$	
			(33) $\bar{y} + \frac{1}{4}, \bar{z} + \frac{1}{4}, \bar{x} + \frac{1}{4}$	(34) $y + \frac{3}{4}, \bar{z} + \frac{3}{4}, x + \frac{1}{4}$	(35) $\bar{y} + \frac{3}{4}, z + \frac{1}{4}, x + \frac{3}{4}$	(36) $y + \frac{1}{4}, z + \frac{3}{4}, \bar{x} + \frac{3}{4}$	
			(37) $\bar{y} + \frac{1}{2}, \bar{x}, z + \frac{1}{2}$	(38) $y, x, z$	(39) $\bar{y}, x + \frac{1}{2}, \bar{z} + \frac{1}{2}$	(40) $y + \frac{1}{2}, \bar{x} + \frac{1}{2}, \bar{z}$	
			(41) $\bar{x} + \frac{1}{2}, \bar{z}, y + \frac{1}{2}$	(42) $x + \frac{1}{2}, \bar{z} + \frac{1}{2}, \bar{y}$	(43) $x, z, y$	(44) $\bar{x}, z + \frac{1}{2}, \bar{y} + \frac{1}{2}$	
			(45) $\bar{z} + \frac{1}{2}, \bar{y}, x + \frac{1}{2}$	(46) $\bar{z}, y + \frac{1}{2}, \bar{x} + \frac{1}{2}$	(47) $z + \frac{1}{2}, \bar{y} + \frac{1}{2}, \bar{x}$	(48) $z, y, x$	

Special: as above, plus

no extra conditions

96	$h$	..2	$\frac{1}{8}, y, \bar{y} + \frac{1}{4}$	$\frac{7}{8}, \bar{y} + \frac{1}{2}, \bar{y} + \frac{3}{4}$	$\frac{3}{8}, y + \frac{1}{2}, y + \frac{3}{4}$	$\frac{5}{8}, \bar{y}, y + \frac{1}{4}$
			$\bar{y} + \frac{1}{4}, \frac{1}{8}, y$	$\bar{y} + \frac{3}{4}, \frac{7}{8}, \bar{y} + \frac{1}{2}$	$y + \frac{3}{4}, \frac{3}{8}, y + \frac{1}{2}$	$y + \frac{1}{4}, \frac{5}{8}, \bar{y}$
			$y, \bar{y} + \frac{1}{4}, \frac{1}{8}$	$\bar{y} + \frac{1}{2}, \bar{y} + \frac{3}{4}, \frac{7}{8}$	$y + \frac{1}{2}, y + \frac{3}{4}, \frac{3}{8}$	$\bar{y}, y + \frac{1}{4}, \frac{5}{8}$
			$\frac{1}{8}, \bar{y} + \frac{1}{4}, y$	$\frac{3}{8}, y + \frac{3}{4}, y + \frac{1}{2}$	$\frac{7}{8}, \bar{y} + \frac{3}{4}, \bar{y} + \frac{1}{2}$	$\frac{5}{8}, y + \frac{1}{4}, \bar{y}$
			$y, \frac{1}{8}, \bar{y} + \frac{1}{4}$	$y + \frac{1}{2}, \frac{3}{8}, y + \frac{3}{4}$	$\bar{y} + \frac{1}{2}, \frac{7}{8}, \bar{y} + \frac{3}{4}$	$\bar{y}, \frac{5}{8}, y + \frac{1}{4}$
			$\bar{y} + \frac{1}{4}, y, \frac{1}{8}$	$y + \frac{3}{4}, y + \frac{1}{2}, \frac{3}{8}$	$\bar{y} + \frac{3}{4}, \bar{y} + \frac{1}{2}, \frac{7}{8}$	$y + \frac{1}{4}, \bar{y}, \frac{5}{8}$

96	$g$	..m	$x, x, z$	$\bar{x}, \bar{x} + \frac{1}{2}, z + \frac{1}{2}$	$\bar{x} + \frac{1}{2}, x + \frac{1}{2}, \bar{z}$	$x + \frac{1}{2}, \bar{x}, \bar{z} + \frac{1}{2}$	no extra conditions
			$z, x, x$	$z + \frac{1}{2}, \bar{x}, \bar{x} + \frac{1}{2}$	$\bar{z}, \bar{x} + \frac{1}{2}, x + \frac{1}{2}$	$\bar{z} + \frac{1}{2}, x + \frac{1}{2}, \bar{x}$	
			$x, z, x$	$\bar{x} + \frac{1}{2}, z + \frac{1}{2}, \bar{x}$	$x + \frac{1}{2}, \bar{z}, \bar{x} + \frac{1}{2}$	$\bar{x}, \bar{z} + \frac{1}{2}, x + \frac{1}{2}$	
			$x + \frac{3}{4}, x + \frac{1}{4}, \bar{z} + \frac{3}{4}$	$\bar{x} + \frac{1}{4}, \bar{x} + \frac{1}{4}, \bar{z} + \frac{1}{4}$	$x + \frac{1}{4}, \bar{x} + \frac{3}{4}, z + \frac{3}{4}$	$\bar{x} + \frac{3}{4}, x + \frac{3}{4}, z + \frac{1}{4}$	
			$x + \frac{3}{4}, z + \frac{1}{4}, \bar{x} + \frac{3}{4}$	$\bar{x} + \frac{3}{4}, z + \frac{3}{4}, x + \frac{1}{4}$	$\bar{x} + \frac{1}{4}, \bar{z} + \frac{1}{4}, \bar{x} + \frac{1}{4}$	$x + \frac{1}{4}, \bar{z} + \frac{3}{4}, x + \frac{3}{4}$	
			$z + \frac{3}{4}, x + \frac{1}{4}, \bar{x} + \frac{3}{4}$	$z + \frac{1}{4}, \bar{x} + \frac{3}{4}, x + \frac{3}{4}$	$\bar{z} + \frac{3}{4}, x + \frac{3}{4}, x + \frac{1}{4}$	$\bar{z} + \frac{1}{4}, \bar{x} + \frac{1}{4}, \bar{x} + \frac{1}{4}$	

48	$f$	2. $mm$	$x, 0, 0$	$\bar{x}, \frac{1}{2}, \frac{1}{2}$	$0, x, 0$	$\frac{1}{2}, \bar{x}, \frac{1}{2}$	$0, 0, x$	$\frac{1}{2}, \frac{1}{2}, \bar{x}$	$hkl$ : $h = 2n + 1$ or $h + k + l = 4n$
			$\frac{3}{4}, x + \frac{1}{4}, \frac{3}{4}$	$\frac{1}{4}, \bar{x} + \frac{1}{4}, \frac{1}{4}$	$x + \frac{3}{4}, \frac{1}{4}, \frac{3}{4}$	$\bar{x} + \frac{3}{4}, \frac{3}{4}, \frac{1}{4}$	$\frac{3}{4}, \frac{1}{4}, \bar{x} + \frac{3}{4}$	$\frac{1}{4}, \frac{3}{4}, x + \frac{3}{4}$	

32	$e$	. $3m$	$x, x, x$	$\bar{x}, \bar{x} + \frac{1}{2}, x + \frac{1}{2}$				no extra conditions
			$\bar{x} + \frac{1}{2}, x + \frac{1}{2}, \bar{x}$	$x + \frac{1}{2}, \bar{x}, \bar{x} + \frac{1}{2}$				
			$x + \frac{3}{4}, x + \frac{1}{4}, \bar{x} + \frac{3}{4}$	$\bar{x} + \frac{1}{4}, \bar{x} + \frac{1}{4}, \bar{x} + \frac{1}{4}$				
			$x + \frac{1}{4}, \bar{x} + \frac{3}{4}, x + \frac{3}{4}$	$\bar{x} + \frac{3}{4}, x + \frac{3}{4}, x + \frac{1}{4}$				

16	$d$	. $\bar{3}m$	$\frac{5}{8}, \frac{5}{8}, \frac{5}{8}$	$\frac{3}{8}, \frac{7}{8}, \frac{1}{8}$	$\frac{7}{8}, \frac{1}{8}, \frac{3}{8}$	$\frac{1}{8}, \frac{3}{8}, \frac{7}{8}$	} $hkl$ : $h = 2n + 1$ or $h, k, l = 4n + 2$ or $h, k, l = 4n$
16	$c$	. $\bar{3}m$	$\frac{1}{8}, \frac{1}{8}, \frac{1}{8}$	$\frac{7}{8}, \frac{3}{8}, \frac{5}{8}$	$\frac{3}{8}, \frac{5}{8}, \frac{7}{8}$	$\frac{5}{8}, \frac{7}{8}, \frac{3}{8}$	

8	$b$	$\bar{4}3m$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	$\frac{1}{4}, \frac{3}{4}, \frac{1}{4}$	} $hkl$ : $h = 2n + 1$ or $h + k + l = 4n$
8	$a$	$\bar{4}3m$	$0, 0, 0$	$\frac{3}{4}, \frac{1}{4}, \frac{3}{4}$	

**Symmetry of special projections**Along [001]  $p4mm$ 

$$\mathbf{a}' = \frac{1}{4}(\mathbf{a} - \mathbf{b}) \quad \mathbf{b}' = \frac{1}{4}(\mathbf{a} + \mathbf{b})$$

Origin at  $0, 0, z$ Along [111]  $p6mm$ 

$$\mathbf{a}' = \frac{1}{6}(2\mathbf{a} - \mathbf{b} - \mathbf{c}) \quad \mathbf{b}' = \frac{1}{6}(-\mathbf{a} + 2\mathbf{b} - \mathbf{c})$$

Origin at  $x, x, x$ Along [110]  $c2mm$ 

$$\mathbf{a}' = \frac{1}{2}(-\mathbf{a} + \mathbf{b}) \quad \mathbf{b}' = \mathbf{c}$$

Origin at  $x, x, \frac{1}{8}$

ORIGIN CHOICE 1

**Maximal non-isomorphic subgroups**

<b>I</b>	[2] $F \bar{4} 3 m$ (216)	(1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; 37; 38; 39; 40; 41; 42; 43; 44; 45; 46; 47; 48)+
	[2] $F 4_1 3 2$ (210)	(1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; 13; 14; 15; 16; 17; 18; 19; 20; 21; 22; 23; 24)+
	[2] $F d \bar{3} 1$ ( $F d \bar{3}$ , 203)	(1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; 25; 26; 27; 28; 29; 30; 31; 32; 33; 34; 35; 36)+
	{ [3] $F 4_1/d 1 2/m$ ( $I 4_1/am d$ , 141)	(1; 2; 3; 4; 13; 14; 15; 16; 25; 26; 27; 28; 37; 38; 39; 40)+
	{ [3] $F 4_1/d 1 2/m$ ( $I 4_1/am d$ , 141)	(1; 2; 3; 4; 17; 18; 19; 20; 25; 26; 27; 28; 41; 42; 43; 44)+
	{ [3] $F 4_1/d 1 2/m$ ( $I 4_1/am d$ , 141)	(1; 2; 3; 4; 21; 22; 23; 24; 25; 26; 27; 28; 45; 46; 47; 48)+
	{ [4] $F 1 \bar{3} 2/m$ ( $R \bar{3} m$ , 166)	(1; 5; 9; 14; 19; 24; 25; 29; 33; 38; 43; 48)+
	{ [4] $F 1 \bar{3} 2/m$ ( $R \bar{3} m$ , 166)	(1; 6; 12; 13; 18; 24; 25; 30; 36; 37; 42; 48)+
	{ [4] $F 1 \bar{3} 2/m$ ( $R \bar{3} m$ , 166)	(1; 7; 10; 13; 19; 22; 25; 31; 34; 37; 43; 46)+
	{ [4] $F 1 \bar{3} 2/m$ ( $R \bar{3} m$ , 166)	(1; 8; 11; 14; 18; 22; 25; 32; 35; 38; 42; 46)+

**IIa** none

**IIb** none

**Maximal isomorphic subgroups of lowest index**

**IIc** [27]  $F d \bar{3} m$  ( $\mathbf{a}' = 3\mathbf{a}, \mathbf{b}' = 3\mathbf{b}, \mathbf{c}' = 3\mathbf{c}$ ) (227)

**Minimal non-isomorphic supergroups**

**I** none

**II** [2]  $P n \bar{3} m$  ( $\mathbf{a}' = \frac{1}{2}\mathbf{a}, \mathbf{b}' = \frac{1}{2}\mathbf{b}, \mathbf{c}' = \frac{1}{2}\mathbf{c}$ ) (224)

**Symmetry operations**

For (0,0,0)+ set

- |   |   |  |   |
|---|---|--|---|
| (1) 1   | (2) $2(0,0,\frac{1}{2})$ $0,\frac{1}{4},z$  | (3) $2(0,\frac{1}{2},0)$ $\frac{1}{4},y,0$   | (4) $2(\frac{1}{2},0,0)$ $x,0,\frac{1}{4}$  |
| (5) $3^+ x,x,x$   | (6) $3^+(\frac{1}{3},-\frac{1}{3},\frac{1}{3})$ $\bar{x}+\frac{1}{6},x+\frac{1}{6},\bar{x}$ | (7) $3^+(-\frac{1}{3},\frac{1}{3},\frac{1}{3})$ $x+\frac{1}{3},\bar{x}-\frac{1}{6},\bar{x}$        | (8) $3^+(\frac{1}{3},\frac{1}{3},-\frac{1}{3})$ $\bar{x}+\frac{1}{6},\bar{x}+\frac{1}{3},x$ |
| (9) $3^- x,x,x$   | (10) $3^- x,\bar{x}+\frac{1}{2},\bar{x}$  | (11) $3^- \bar{x}+\frac{1}{2},\bar{x},x$   | (12) $3^- \bar{x}-\frac{1}{2},x+\frac{1}{2},\bar{x}$  |
| (13) $2(\frac{1}{2},\frac{1}{2},0)$ $x,x-\frac{1}{4},\frac{3}{8}$                   | (14) $2 x,\bar{x}+\frac{1}{4},\frac{1}{8}$  | (15) $4^-(0,0,\frac{3}{4})$ $\frac{1}{2},\frac{1}{4},z$  | (16) $4^+(0,0,\frac{1}{4})$ $0,\frac{3}{4},z$   |
| (17) $4^-(\frac{3}{4},0,0)$ $x,\frac{1}{2},\frac{1}{4}$                             | (18) $2(0,\frac{1}{2},\frac{1}{2})$ $\frac{3}{8},y+\frac{1}{4},y$                           | (19) $2 \frac{1}{8},y+\frac{1}{4},\bar{y}$   | (20) $4^+(\frac{1}{4},0,0)$ $x,0,\frac{3}{4}$   |
| (21) $4^+(0,\frac{1}{4},0)$ $\frac{3}{4},y,0$                                       | (22) $2(\frac{1}{2},0,\frac{1}{2})$ $x-\frac{1}{4},\frac{3}{8},x$                           | (23) $4^-(0,\frac{3}{4},0)$ $\frac{1}{4},y,\frac{1}{2}$  | (24) $2 \bar{x}+\frac{1}{4},\frac{1}{8},x$  |
| (25) $\bar{1} \frac{1}{8},\frac{1}{8},\frac{1}{8}$                                  | (26) $d(\frac{1}{4},\frac{3}{4},0)$ $x,y,\frac{3}{8}$                                       | (27) $d(\frac{3}{4},0,\frac{1}{4})$ $x,\frac{3}{8},z$  | (28) $d(0,\frac{1}{4},\frac{3}{4})$ $\frac{3}{8},y,z$                                       |
| (29) $\bar{3}^+ x,x,x;$ $\frac{1}{8},\frac{1}{8},\frac{1}{8}$                       | (30) $\bar{3}^+ \bar{x}-1,x+1,\bar{x};$ $-\frac{1}{8},\frac{1}{8},\frac{7}{8}$              | (31) $\bar{3}^+ x,\bar{x}+1,\bar{x};$ $\frac{1}{8},\frac{7}{8},-\frac{1}{8}$                       | (32) $\bar{3}^+ \bar{x}+1,\bar{x},x;$ $\frac{7}{8},-\frac{1}{8},\frac{1}{8}$                |
| (33) $\bar{3}^- x,x,x;$ $\frac{1}{8},\frac{1}{8},\frac{1}{8}$                       | (34) $\bar{3}^- x+\frac{3}{2},\bar{x}-1,\bar{x};$ $\frac{5}{8},-\frac{1}{8},\frac{7}{8}$    | (35) $\bar{3}^- \bar{x}+\frac{1}{2},\bar{x}+\frac{3}{2},x;$ $-\frac{1}{8},\frac{7}{8},\frac{5}{8}$ | (36) $\bar{3}^- \bar{x}+1,x+\frac{1}{2},\bar{x};$ $\frac{7}{8},\frac{5}{8},-\frac{1}{8}$    |
| (37) $g(\frac{1}{4},-\frac{1}{4},\frac{1}{2})$ $x+\frac{1}{4},\bar{x},z$            | (38) $m x,x,z$  | (39) $\bar{4}^- -\frac{1}{4},\frac{1}{4},z;$ $-\frac{1}{4},\frac{1}{4},\frac{1}{4}$                | (40) $\bar{4}^+ \frac{1}{2},0,z;$ $\frac{1}{2},0,0$   |
| (41) $\bar{4}^- x,-\frac{1}{4},\frac{1}{4};$ $\frac{1}{4},-\frac{1}{4},\frac{1}{4}$ | (42) $g(\frac{1}{2},\frac{1}{4},-\frac{1}{4})$ $x,y+\frac{1}{4},\bar{y}$                    | (43) $m x,y,y$   | (44) $\bar{4}^+ x,\frac{1}{2},0;$ $0,\frac{1}{2},0$   |
| (45) $\bar{4}^+ 0,y,\frac{1}{2};$ $0,0,\frac{1}{2}$                                 | (46) $g(-\frac{1}{4},\frac{1}{2},\frac{1}{4})$ $\bar{x}+\frac{1}{4},y,x$                    | (47) $\bar{4}^- \frac{1}{4},y,-\frac{1}{4};$ $\frac{1}{4},\frac{1}{4},-\frac{1}{4}$                | (48) $m x,y,x$  |

For (0, $\frac{1}{2},\frac{1}{2}$ )+ set

- |   |  |  |  |
|---|--|--|--|
| (1) $t(0,\frac{1}{2},\frac{1}{2})$  | (2) 2 0,0,z  | (3) 2 $\frac{1}{4},y,\frac{1}{4}$  | (4) $2(\frac{1}{2},0,0)$ $x,\frac{1}{4},0$   |
| (5) $3^+(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ $x-\frac{1}{3},x-\frac{1}{6},x$        | (6) $3^+ \bar{x}+\frac{1}{2},x,\bar{x}$  | (7) $3^+ x,\bar{x},\bar{x}$  | (8) $3^+ \bar{x}+\frac{1}{2},\bar{x}+\frac{1}{2},x$                                      |
| (9) $3^-(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ $x-\frac{1}{6},x+\frac{1}{6},x$        | (10) $3^- x+\frac{1}{2},\bar{x},\bar{x}$   | (11) $3^-(\frac{1}{3},\frac{1}{3},-\frac{1}{3})$ $\bar{x}+\frac{1}{3},\bar{x}+\frac{1}{6},x$ | (12) $3^- \bar{x},x,\bar{x}$   |
| (13) $2(\frac{3}{4},\frac{3}{4},0)$ $x,x,\frac{1}{8}$                                 | (14) $2(-\frac{1}{4},\frac{1}{4},0)$ $x,\bar{x}+\frac{1}{2},\frac{3}{8}$                 | (15) $4^-(0,0,\frac{1}{4})$ $\frac{1}{4},0,z$  | (16) $4^+(0,0,\frac{3}{4})$ $\frac{1}{4},\frac{1}{2},z$                                  |
| (17) $4^-(\frac{3}{4},0,0)$ $x,\frac{1}{2},-\frac{1}{4}$                              | (18) $2(0,\frac{1}{2},\frac{1}{2})$ $\frac{3}{8},y-\frac{1}{4},y$                        | (19) $2 \frac{1}{8},y+\frac{3}{4},\bar{y}$   | (20) $4^+(\frac{1}{4},0,0)$ $x,0,\frac{1}{4}$  |
| (21) $4^+(0,\frac{3}{4},0)$ $\frac{1}{2},y,-\frac{1}{4}$                              | (22) $2(\frac{1}{4},0,\frac{1}{4})$ $x,\frac{1}{8},x$                                    | (23) $4^-(0,\frac{1}{4},0)$ $0,y,\frac{3}{4}$  | (24) $2(-\frac{1}{4},0,\frac{1}{4})$ $\bar{x}+\frac{1}{2},\frac{3}{8},x$                 |
| (25) $\bar{1} \frac{1}{8},\frac{3}{8},\frac{3}{8}$                                    | (26) $d(\frac{1}{4},\frac{1}{4},0)$ $x,y,\frac{1}{8}$                                    | (27) $d(\frac{3}{4},0,\frac{3}{4})$ $x,\frac{1}{8},z$  | (28) $d(0,\frac{3}{4},\frac{1}{4})$ $\frac{3}{8},y,z$                                    |
| (29) $\bar{3}^+ x,x+\frac{1}{2},x;$ $\frac{1}{8},\frac{5}{8},\frac{1}{8}$             | (30) $\bar{3}^+ \bar{x}-1,x+\frac{3}{2},\bar{x};$ $-\frac{1}{8},\frac{5}{8},\frac{7}{8}$ | (31) $\bar{3}^+ x,\bar{x}+\frac{1}{2},\bar{x};$ $\frac{1}{8},\frac{3}{8},-\frac{1}{8}$       | (32) $\bar{3}^+ \bar{x}+1,\bar{x}-\frac{1}{2},x;$ $\frac{7}{8},-\frac{5}{8},\frac{1}{8}$ |
| (33) $\bar{3}^- x-\frac{1}{2},x-\frac{1}{2},x;$ $\frac{1}{8},\frac{1}{8},\frac{5}{8}$ | (34) $\bar{3}^- x+1,\bar{x}-\frac{3}{2},\bar{x};$ $\frac{1}{8},-\frac{5}{8},\frac{7}{8}$ | (35) $\bar{3}^- \bar{x},\bar{x}+1,x;$ $-\frac{1}{8},\frac{7}{8},\frac{1}{8}$                 | (36) $\bar{3}^- \bar{x}+\frac{1}{2},x,\bar{x};$ $\frac{3}{8},\frac{1}{8},-\frac{1}{8}$   |
| (37) $m x+\frac{1}{2},\bar{x},z$  | (38) $g(\frac{1}{4},\frac{1}{4},\frac{1}{2})$ $x-\frac{1}{4},x,z$                        | (39) $\bar{4}^- 0,0,z;$ $0,0,0$  | (40) $\bar{4}^+ \frac{1}{4},-\frac{1}{4},z;$ $\frac{1}{4},-\frac{1}{4},\frac{1}{4}$      |
| (41) $\bar{4}^- x,\frac{1}{4},\frac{1}{4};$ $\frac{1}{4},\frac{1}{4},\frac{1}{4}$     | (42) $g(\frac{1}{2},-\frac{1}{4},\frac{1}{4})$ $x,y+\frac{1}{4},\bar{y}$                 | (43) $g(0,\frac{1}{2},\frac{1}{2})$ $x,y,y$  | (44) $\bar{4}^+ x,0,0;$ $0,0,0$  |
| (45) $\bar{4}^+ \frac{1}{4},y,\frac{1}{4};$ $\frac{1}{4},\frac{1}{4},\frac{1}{4}$     | (46) $m \bar{x},y,x$   | (47) $\bar{4}^- \frac{1}{2},y,0;$ $\frac{1}{2},0,0$  | (48) $g(\frac{1}{4},\frac{1}{2},\frac{1}{4})$ $x-\frac{1}{4},y,x$                        |

For ( $\frac{1}{2},0,\frac{1}{2}$ )+ set

- |   |  |  |  |
|---|--|--|--|
| (1) $t(\frac{1}{2},0,\frac{1}{2})$  | (2) 2 $\frac{1}{4},\frac{1}{4},z$  | (3) $2(0,\frac{1}{2},0)$ $0,y,\frac{1}{4}$   | (4) 2 $x,0,0$  |
| (5) $3^+(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ $x+\frac{1}{6},x-\frac{1}{6},x$        | (6) $3^+ \bar{x},x,\bar{x}$  | (7) $3^+ x+\frac{1}{2},\bar{x},\bar{x}$  | (8) $3^+ \bar{x},\bar{x}+\frac{1}{2},x$  |
| (9) $3^-(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ $x-\frac{1}{6},x-\frac{1}{3},x$        | (10) $3^-(-\frac{1}{3},\frac{1}{3},\frac{1}{3})$ $x+\frac{1}{6},\bar{x}+\frac{1}{6},\bar{x}$       | (11) $3^- \bar{x},\bar{x},x$   | (12) $3^- \bar{x},x+\frac{1}{2},\bar{x}$   |
| (13) $2(\frac{1}{4},\frac{1}{4},0)$ $x,x,\frac{1}{8}$                                 | (14) $2(\frac{1}{4},-\frac{1}{4},0)$ $x,\bar{x}+\frac{1}{2},\frac{3}{8}$                           | (15) $4^-(0,0,\frac{1}{4})$ $\frac{3}{4},0,z$  | (16) $4^+(0,0,\frac{3}{4})$ $-\frac{1}{4},\frac{1}{2},z$   |
| (17) $4^-(\frac{1}{4},0,0)$ $x,\frac{1}{4},0$   | (18) $2(0,\frac{3}{4},\frac{3}{4})$ $\frac{1}{8},y,y$  | (19) $2(0,-\frac{1}{4},\frac{1}{4})$ $\frac{3}{8},y+\frac{1}{2},\bar{y}$                           | (20) $4^+(\frac{3}{4},0,0)$ $x,\frac{1}{4},\frac{1}{2}$  |
| (21) $4^+(0,\frac{1}{4},0)$ $\frac{1}{4},y,0$   | (22) $2(\frac{1}{2},0,\frac{1}{2})$ $x+\frac{1}{4},\frac{3}{8},x$                                  | (23) $4^-(0,\frac{3}{4},0)$ $-\frac{1}{4},y,\frac{1}{2}$   | (24) $2 \bar{x}+\frac{3}{4},\frac{1}{8},x$   |
| (25) $\bar{1} \frac{3}{8},\frac{1}{8},\frac{3}{8}$                                    | (26) $d(\frac{3}{4},\frac{3}{4},0)$ $x,y,\frac{1}{8}$  | (27) $d(\frac{1}{4},0,\frac{3}{4})$ $x,\frac{3}{8},z$  | (28) $d(0,\frac{1}{4},\frac{1}{4})$ $\frac{1}{8},y,z$  |
| (29) $\bar{3}^+ x-\frac{1}{2},x-\frac{1}{2},x;$ $\frac{1}{8},\frac{1}{8},\frac{5}{8}$ | (30) $\bar{3}^+ \bar{x}-\frac{1}{2},x+\frac{1}{2},\bar{x};$ $-\frac{1}{8},\frac{1}{8},\frac{3}{8}$ | (31) $\bar{3}^+ x-\frac{1}{2},\bar{x}+\frac{3}{2},\bar{x};$ $\frac{1}{8},\frac{7}{8},-\frac{5}{8}$ | (32) $\bar{3}^+ \bar{x}+\frac{3}{2},\bar{x}+\frac{1}{2},x;$ $\frac{7}{8},-\frac{1}{8},\frac{5}{8}$ |
| (33) $\bar{3}^- x+\frac{1}{2},x,x;$ $\frac{5}{8},\frac{1}{8},\frac{1}{8}$             | (34) $\bar{3}^- x+1,\bar{x}-1,\bar{x};$ $\frac{1}{8},-\frac{1}{8},\frac{7}{8}$                     | (35) $\bar{3}^- \bar{x},\bar{x}+\frac{1}{2},x;$ $-\frac{1}{8},\frac{3}{8},\frac{1}{8}$             | (36) $\bar{3}^- \bar{x}+\frac{3}{2},x-\frac{1}{2},\bar{x};$ $\frac{7}{8},\frac{1}{8},-\frac{5}{8}$ |
| (37) $m x,\bar{x},z$  | (38) $g(\frac{1}{4},\frac{1}{4},\frac{1}{2})$ $x+\frac{1}{4},x,z$                                  | (39) $\bar{4}^- 0,\frac{1}{2},z;$ $0,\frac{1}{2},0$  | (40) $\bar{4}^+ \frac{1}{4},\frac{1}{4},z;$ $\frac{1}{4},\frac{1}{4},\frac{1}{4}$                  |
| (41) $\bar{4}^- x,0,0;$ $0,0,0$   | (42) $m x,y+\frac{1}{2},\bar{y}$   | (43) $g(\frac{1}{2},\frac{1}{4},\frac{1}{4})$ $x,y-\frac{1}{4},y$                                  | (44) $\bar{4}^+ x,\frac{1}{4},-\frac{1}{4};$ $\frac{1}{4},\frac{1}{4},-\frac{1}{4}$                |
| (45) $\bar{4}^+ 0,y,0;$ $0,0,0$   | (46) $g(\frac{1}{4},\frac{1}{2},-\frac{1}{4})$ $\bar{x}+\frac{1}{4},y,x$                           | (47) $\bar{4}^- \frac{1}{4},y,\frac{1}{4};$ $\frac{1}{4},\frac{1}{4},\frac{1}{4}$                  | (48) $g(\frac{1}{2},0,\frac{1}{2})$ $x,y,x$  |

For ( $\frac{1}{2},\frac{1}{2},0$ )+ set

- |   |  |  |  |
|---|--|--|--|
| (1) $t(\frac{1}{2},\frac{1}{2},0)$  | (2) $2(0,0,\frac{1}{2})$ $\frac{1}{4},0,z$   | (3) 2 0,y,0  | (4) 2 $x,\frac{1}{4},\frac{1}{4}$  |
| (5) $3^+(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ $x+\frac{1}{6},x+\frac{1}{3},x$      | (6) $3^+ \bar{x},x+\frac{1}{2},\bar{x}$  | (7) $3^+ x+\frac{1}{2},\bar{x}-\frac{1}{2},\bar{x}$                                      | (8) $3^+ \bar{x},\bar{x},x$  |
| (9) $3^-(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ $x+\frac{1}{3},x+\frac{1}{6},x$      | (10) $3^- x,\bar{x},\bar{x}$   | (11) $3^- \bar{x}+\frac{1}{2},\bar{x}+\frac{1}{2},x$                                     | (12) $3^-(\frac{1}{3},-\frac{1}{3},\frac{1}{3})$ $\bar{x}-\frac{1}{6},x+\frac{1}{3},\bar{x}$ |
| (13) $2(\frac{1}{2},\frac{1}{2},0)$ $x,x+\frac{1}{4},\frac{3}{8}$                   | (14) $2 x,\bar{x}+\frac{3}{4},\frac{1}{8}$   | (15) $4^-(0,0,\frac{3}{4})$ $\frac{1}{2},-\frac{1}{4},z$                                 | (16) $4^+(0,0,\frac{1}{4})$ $0,\frac{1}{4},z$  |
| (17) $4^-(\frac{1}{4},0,0)$ $x,\frac{3}{4},0$                                       | (18) $2(0,\frac{1}{4},\frac{1}{4})$ $\frac{1}{8},y,y$  | (19) $2(0,\frac{1}{4},-\frac{1}{4})$ $\frac{3}{8},y+\frac{1}{2},\bar{y}$                 | (20) $4^+(\frac{3}{4},0,0)$ $x,-\frac{1}{4},\frac{1}{2}$                                     |
| (21) $4^+(0,\frac{3}{4},0)$ $\frac{1}{2},y,\frac{1}{4}$                             | (22) $2(\frac{3}{4},0,\frac{3}{4})$ $x,\frac{1}{8},x$  | (23) $4^-(0,\frac{1}{4},0)$ $0,y,\frac{1}{4}$  | (24) $2(\frac{1}{4},0,-\frac{1}{4})$ $\bar{x}+\frac{1}{2},\frac{3}{8},x$                     |
| (25) $\bar{1} \frac{3}{8},\frac{3}{8},\frac{1}{8}$                                  | (26) $d(\frac{3}{4},\frac{1}{4},0)$ $x,y,\frac{3}{8}$  | (27) $d(\frac{1}{4},0,\frac{1}{4})$ $x,\frac{1}{8},z$                                    | (28) $d(0,\frac{3}{4},\frac{3}{4})$ $\frac{1}{8},y,z$  |
| (29) $\bar{3}^+ x+\frac{1}{2},x,x;$ $\frac{5}{8},\frac{1}{8},\frac{1}{8}$           | (30) $\bar{3}^+ \bar{x}-\frac{3}{2},x+1,\bar{x};$ $-\frac{5}{8},\frac{1}{8},\frac{7}{8}$           | (31) $\bar{3}^+ x+\frac{1}{2},\bar{x}+1,\bar{x};$ $\frac{5}{8},\frac{7}{8},-\frac{1}{8}$ | (32) $\bar{3}^+ \bar{x}+\frac{1}{2},\bar{x},x;$ $\frac{3}{8},-\frac{1}{8},\frac{1}{8}$       |
| (33) $\bar{3}^- x,x+\frac{1}{2},x;$ $\frac{1}{8},\frac{5}{8},\frac{1}{8}$           | (34) $\bar{3}^- x+\frac{1}{2},\bar{x}-\frac{1}{2},\bar{x};$ $\frac{1}{8},-\frac{1}{8},\frac{3}{8}$ | (35) $\bar{3}^- \bar{x}-\frac{1}{2},\bar{x}+1,x;$ $-\frac{5}{8},\frac{7}{8},\frac{1}{8}$ | (36) $\bar{3}^- \bar{x}+1,x,\bar{x};$ $\frac{7}{8},\frac{1}{8},-\frac{1}{8}$                 |
| (37) $g(-\frac{1}{4},\frac{1}{4},\frac{1}{2})$ $x+\frac{1}{4},\bar{x},z$            | (38) $g(\frac{1}{2},\frac{1}{2},0)$ $x,x,z$  | (39) $\bar{4}^- \frac{1}{4},\frac{1}{4},z;$ $\frac{1}{4},\frac{1}{4},\frac{1}{4}$        | (40) $\bar{4}^+ 0,0,z;$ $0,0,0$  |
| (41) $\bar{4}^- x,0,\frac{1}{2};$ $0,0,\frac{1}{2}$                                 | (42) $m x,y,\bar{y}$   | (43) $g(\frac{1}{2},\frac{1}{4},\frac{1}{4})$ $x,y+\frac{1}{4},y$                        | (44) $\bar{4}^+ x,\frac{1}{4},\frac{1}{4};$ $\frac{1}{4},\frac{1}{4},\frac{1}{4}$            |
| (45) $\bar{4}^+ -\frac{1}{4},y,\frac{1}{4};$ $-\frac{1}{4},\frac{1}{4},\frac{1}{4}$ | (46) $m \bar{x}+\frac{1}{2},y,x$   | (47) $\bar{4}^- 0,y,0;$ $0,0,0$  | (48) $g(\frac{1}{4},\frac{1}{2},\frac{1}{4})$ $x+\frac{1}{4},y,x$                            |