

2.1. STATISTICAL PROPERTIES OF THE WEIGHTED RECIPROCAL LATTICE

denominators, but not in the numerators, of equation (2.1.2.6) by their arguments, and this is equivalent to the approximation $\sin x \simeq x$ in the denominators only. This is a good approximation for atoms close together in the structure and thus giving the largest terms in the sums in equation (2.1.2.6), and gives the correct sign and order of magnitude even for x having its maximum value of $\pi/2$.

2.1.2.2. Physical background

The preceding section has used mathematical arguments. From a physical point of view, the radiation diffracted by atoms that are resolved will interfere destructively, so that the resulting intensity will be the sum of the intensities diffracted by individual atoms, whereas that from completely unresolved atoms will interfere constructively, so that *amplitudes* rather than intensities add. In intermediate cases there will be partial constructive interference. Resolution in accordance with the Rayleigh (1879) criterion requires that $s = (2 \sin \theta)/\lambda$ should be greater than half the reciprocal of the minimum interatomic distance in the crystal (Wilson, 1979); full resolution requires a substantial multiple of this. This criterion is essentially equivalent to that proposed from the study of a special case of the second process in the preceding section.

2.1.2.3. An approximation for organic compounds

In organic compounds there are very many interatomic distances of about 1.5 or 1.4 Å. Adoption of the preceding criterion would mean that the inner portion of the region of reciprocal space accessible by the use of copper $K\alpha$ radiation is not within the sphere of intensity statistics based on fixed-index (first process) averaging. No substantial results are available for fixed-parameter (second process) averaging, and very few from the approximation to it (third process).

To the extent to which the third process is acceptable, an approximation to the variation of $\langle I \rangle$ with $\sin \theta$ is obtainable. The exponent in equation (2.1.2.2) can be written as

$$2\pi i s r_{jk} \cos \psi, \quad (2.1.2.9)$$

where s is the radial distance in reciprocal space, r_{jk} is the distance from the j th to the k th atom and ψ is the angle between the vectors s and r . Averaging over a sphere of radius s , with ψ treated as the co-latitude, gives

$$\langle I \rangle = \sum_j \sum_k f_j f_k \frac{\sin 2\pi s r_{jk}}{2\pi s r_{jk}}. \quad (2.1.2.10)$$

This is the familiar Debye expression. It has the correct limits for s zero and s large, and is in accord with the argument from resolution.

2.1.2.4. Effect of centring

In the preceding discussion there has been a tacit assumption that the lattice is primitive. A centred crystal can always be referred to a primitive lattice and if this is done no change is required. If the centred lattice is retained, many reflections are identically zero and the intensity of the non-zero reflections is enhanced by a factor of two (I and C lattices) or four (F lattice), so that the average intensity of all the reflections, zero and non-zero taken together, is unchanged.

Other symmetry elements affect only zones and rows of reflections, and so do not affect the general average when the total number of reflections is large. Their effect on zones and rows is discussed in Section 2.1.3.

Table 2.1.3.1. *Intensity-distribution effects of symmetry elements causing systematic absences*

Abbreviations and orientation of axes: A = acentric distribution, C = centric distribution, Z = systematically zero, S = distribution parameter, $\langle I \rangle$ = average intensity. Axes are parallel to c , planes are perpendicular to c .

Element	Reflections	Distribution	S/Σ	$\langle I \rangle/\Sigma$
2_1	hkl	A	1	1
	$hk0$	C	1	1
	$00l$	$(Z + A)/2$	1	2
$3_1, 3_2$	hkl	A	1	1
	$hk0$	A	1	1
	$00l$	$(2Z + A)/3$	1	3
$4_1, 4_3$	hkl	A	1	1
	$hk0$	C	1	1
	$00l$	$(3Z + A)/4$	1	4
4_2	hkl	A	1	1
	$hk0$	C	1	1
	$00l$	$(Z + A)/2$	2	4
$6_1, 6_5$	hkl	A	1	1
	$hk0$	C	1	1
	$00l$	$(5Z + A)/6$	1	6
$6_2, 6_4$	hkl	A	1	1
	$hk0$	C	1	1
	$00l$	$(2Z + A)/3$	2	6
6_3	hkl	A	1	1
	$hk0$	C	1	1
	$00l$	$(Z + A)/2$	3	6
a	hkl	A	1	1
	$hk0$	$(Z + A)/2$	1	2
	$00l$	C	1	1
	$0k0$	A	2	2
C, I	All	$(Z + A)/2$	1	2
F	All	$(3Z + A)/2$	1	4

2.1.3. The average intensity of zones and rows

2.1.3.1. Symmetry elements producing systematic absences

Symmetry elements can be divided into two types: those that cause systematic absences and those that do not. Those producing systematic absences (glide planes and screw axes) produce at the same time groups of reflections (confined to zones and rows in reciprocal space, respectively) with an average intensity an integral* multiple of the general average. The effects for single symmetry elements of this type are given in Table 2.1.3.1 for the general reflections hkl and separately for any zones and rows that are affected. The 'average multipliers' are given in the column headed $\langle I \rangle/\Sigma$; 'distribution' and 'distribution parameters' are treated in Section 2.1.5. As for the centring, the fraction of reflections missing and the integer multiplying the average are related in such a way that the overall intensity is unchanged. The

* The multiple is given as an exact integer for fixed-index averaging, an approximate integer for fixed-parameter averaging. Statements should be understood to refer to fixed-index averaging unless the contrary is explicitly stated.