behaviour (Fukuhara, 1966). Explicit solutions for the three-beam case, which displays some aspects of many-beam character, have been obtained (Gjønnes & Høier, 1971; Hurley & Moodie, 1980).

2.5.2.6. *Imaging with electrons*

Electron optics. Electrons may be focused by use of axially symmetric magnetic fields produced by electromagnetic lenses. The focal length of such a lens used as a projector lens (focal points outside the lens field) is given by

$$f_p^{-1} = \frac{e}{8mW_r} \int_{-\infty}^{\infty} H_z^2(z)\, \mathrm{d}z, \qquad (2.5.2.28)$$

where W_r is the relativistically corrected accelerating voltage and H_z is the z component of the magnetic field. An expression in terms of experimental constants was given by Liebman (1955) as

$$\frac{1}{f} = \frac{A_0(NI)^2}{W_r(S+D)}, \qquad (2.5.2.29)$$

where A_0 is a constant, NI is the number of ampere turns of the lens winding, S is the length of the gap between the magnet pole pieces and D is the bore of the pole pieces.

Lenses of this type have irreducible aberrations, the most important of which for the paraxial conditions of electron microscopy is the third-order spherical aberration, coefficient C_s, giving a variation of focal length of $C_s\alpha^2$ for a beam at an angle α to the axis. Chromatic aberration, coefficient C_c, gives a spread of focal lengths

$$\Delta f = C_c\left(\frac{\Delta W_0}{W_0} + 2\frac{\Delta I}{I}\right) \qquad (2.5.2.30)$$

for variations ΔW_0 and ΔI of the accelerating voltage and lens currents, respectively.

The objective lens of an electron microscope is the critical lens for the determination of image resolution and contrast. The action of this lens in a conventional transmission electron microscope (TEM) is described by use of the Abbe theory for coherent incident illumination transmitted through the object to produce a wavefunction $\psi_0(xy)$ (see Fig. 2.5.2.2).

The amplitude distribution in the back focal plane of the objective lens is written

$$\Psi_0(u,v)\cdot T(u,v), \qquad (2.5.2.31)$$

where $\Psi_0(u,v)$ is the Fourier transform of $\psi_0(x,y)$ and $T(u,v)$ is the transfer function of the lens, consisting of an aperture function

$$A(u,v) = \begin{cases} 1 & \text{for } (u^2+v^2)^{1/2} \le A \\ 0 & \text{elsewhere} \end{cases} \qquad (2.5.2.32)$$

and a phase function $\exp\{i\chi(u,v)\}$ where the phase perturbation $\chi(uv)$ due to lens defocus Δf and aberrations is usually approximated as

$$\chi(uv) = \pi\cdot\Delta f\cdot\lambda(u^2+v^2) + \frac{\pi}{2}C_s\lambda^3(u^2+v^2)^2, \qquad (2.5.2.33)$$

and u, v are the reciprocal-space variables related to the scattering angles φ_x, φ_y by

$$u = (\sin\varphi_x)/\lambda,$$
$$v = (\sin\varphi_y)/\lambda.$$

The image amplitude distribution, referred to the object coordinates, is given by Fourier transform of (2.5.2.31) as

$$\psi(xy) = \psi_0(xy) * t(xy), \qquad (2.5.2.34)$$

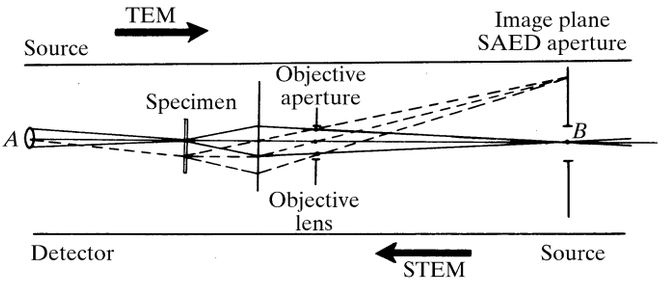

Fig. 2.5.2.2. Diagram representing the critical components of a conventional transmission electron microscope (TEM) and a scanning transmission electron microscope (STEM). For the TEM, electrons from a source A illuminate the specimen and the objective lens forms an image of the transmitted electrons on the image plane, B. For the STEM, a source at B is imaged by the objective lens to form a small probe on the specimen and some part of the transmitted beam is collected by a detector at A.

where $t(xy)$, given by Fourier transform of $T(u,v)$, is the spread function. The image intensity is then

$$I(xy) = |\psi(xy)|^2 = |\psi_0(xy) * t(xy)|^2. \qquad (2.5.2.35)$$

In practice the coherent imaging theory provides a good approximation but limitations of the coherence of the illumination have appreciable effects under high-resolution imaging conditions.

The variation of focal lengths according to (2.5.2.30) is described by a function $G(\Delta f)$. Illumination from a finite incoherent source gives a distribution of incident-beam angles $H(u_1,v_1)$. Then the image intensity is found by integrating incoherently over Δf and u_1, v_1:

$$I(xy) = \int\int G(\Delta f)\cdot H(u_1v_1) \times |\mathscr{F}\{\Psi_0(u-u_1, v-v_1)\cdot T_{\Delta f}(u,v)\}|^2\, \mathrm{d}(\Delta f)\cdot\, \mathrm{d}u_1\, \mathrm{d}v_1, \qquad (2.5.2.36)$$

where $\mathscr{F}$ denotes the Fourier-transform operation.

In the scanning transmission electron microscope (STEM), the objective lens focuses a small bright source of electrons on the object and directly transmitted or scattered electrons are detected to form an image as the incident beam is scanned over the object (see Fig. 2.5.2.2). Ideally the image amplitude can be related to that of the conventional transmission electron microscope by use of the 'reciprocity relationship' which refers to point sources and detectors for scalar radiation in scalar fields with elastic scattering processes only. It may be stated: 'The amplitude at a point B due to a point source at A is identical to that which would be produced at A for the identical source placed at B'.

For an axial point source, the amplitude distribution produced by the objective lens on the specimen is

$$\mathscr{F}[T(u,v)] = t(xy). \qquad (2.5.2.37)$$

If this is translated by the scan to X, Y, the transmitted wave is

$$\psi_0(xy) = q(xy)\cdot t(x-X, y-Y). \qquad (2.5.2.38)$$

The amplitude on the plane of observation following the specimen is then

$$\Psi(uv) = Q(u,v) * \{T(uv)\exp[2\pi i(uX+vY)]\}, \qquad (2.5.2.39)$$

and the image signal produced by a detector having a sensitivity function $H(u, v)$ is

references