

1.2. APPLICATION TO THE CRYSTAL SYSTEMS

$$\begin{aligned} r^{*2} &= (h^2 + k^2 + l^2)a^{*2} + 2(hk + hl + kl)a^{*2} \cos \alpha^* \\ &= s_1 a^{*2} + 2s_2 a^{*2} \cos \alpha^* \end{aligned} \quad (1.1.2.2f)$$

with

$$s_1 = h^2 + k^2 + l^2 \quad \text{and} \quad s_2 = hk + hl + kl.$$

For each value of $s_1 \leq 50$, all corresponding values of s_2 and all triplets h, k, l are listed in Table 1.2.5.2.

$$\frac{u}{h} + \frac{v+w}{h} \cos \alpha = \frac{v}{k} + \frac{u+w}{k} \cos \alpha = \frac{w}{l} + \frac{u+v}{l} \cos \alpha, \quad (1.1.2.12f)$$

$$\begin{aligned} \mathbf{t}_1 \cdot \mathbf{t}_2 &= (u_1 u_2 + v_1 v_2 + w_1 w_2) a^2 \\ &\quad + (u_1 v_2 + u_2 v_1 + u_1 w_2 + u_2 w_1) \\ &\quad + v_1 w_2 + v_2 w_1) a^2 \cos \alpha, \end{aligned} \quad (1.1.3.4f)$$

$$\begin{aligned} \mathbf{r}_1^* \cdot \mathbf{r}_2^* &= (h_1 h_2 + k_1 k_2 + l_1 l_2) a^{*2} \\ &\quad + (h_1 k_2 + h_2 k_1 + h_1 l_2 + h_2 l_1 \\ &\quad + k_1 l_2 + k_2 l_1) a^{*2} \cos \alpha^*. \end{aligned} \quad (1.1.3.7f)$$

Simplified formulae:

$$V = (\mathbf{abc}) = \left[\begin{vmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{vmatrix} \right]^{1/2} = a^3, \quad (1.1.1.1g)$$

$$a^* = b^* = c^* = \frac{1}{a}, \quad \alpha^* = \beta^* = \gamma^* = 90^\circ, \quad (1.1.1.3g)$$

$$V^* = (\mathbf{a}^* \mathbf{b}^* \mathbf{c}^*) = \left[\begin{vmatrix} a^{*2} & 0 & 0 \\ 0 & a^{*2} & 0 \\ 0 & 0 & a^{*2} \end{vmatrix} \right]^{1/2} = a^{*3} = a^{-3}, \quad (1.1.1.4g)$$

$$a = b = c = \frac{1}{a^*}, \quad \alpha = \beta = \gamma = 90^\circ, \quad (1.1.1.7g)$$

$$t^2 = (u^2 + v^2 + w^2) a^2, \quad (1.1.2.1g)$$

$$r^{*2} = (h^2 + k^2 + l^2) a^{*2} = sa^{*2} \quad (1.1.2.2g)$$

with

$$s = h^2 + k^2 + l^2.$$

For each value of $s \leq 100$, all corresponding triplets h, k, l are listed in Table 1.2.6.1.

$$\frac{u}{h} = \frac{v}{k} = \frac{w}{l}, \quad (1.1.2.12g)$$

1.2.6. Cubic crystal system

Metrical conditions:

$$a = b = c; \alpha = \beta = \gamma = 90^\circ$$

Bravais lattice types:

$$cP, cI, cF$$

Symmetry of lattice points:

$$m\bar{3}m$$

$$\mathbf{t}_1 \cdot \mathbf{t}_2 = (u_1 u_2 + v_1 v_2 + w_1 w_2) a^2, \quad (1.1.3.4g)$$

$$\mathbf{r}_1^* \cdot \mathbf{r}_2^* = (h_1 h_2 + k_1 k_2 + l_1 l_2) a^{*2}. \quad (1.1.3.7g)$$

Table 1.2.6.1. Assignment of integers $s \leq 100$ to triplets h, k, l with $s = h^2 + k^2 + l^2$

Each triplet represents all 48 triplets resulting from permutations and sign combinations.

s	$h \ k \ l$								
1	1 0 0	25	5 0 0	42	5 4 1	59	7 3 1	74	8 3 1
2	1 1 0		4 3 0	43	5 3 3		5 5 3		7 5 0
3	1 1 1	26	5 1 0	44	6 2 2	61	6 5 0		7 4 3
4	2 0 0		4 3 1	45	6 3 0		6 4 3	75	7 5 1
5	2 1 0	27	5 1 1		5 4 2	62	7 3 2		5 5 5
6	2 1 1		3 3 3	46	6 3 1		6 5 1	76	6 6 2
8	2 2 0	29	5 2 0	48	4 4 4	64	8 0 0	77	8 3 2
9	3 0 0		4 3 2	49	7 0 0	65	8 1 0		6 5 4
	2 2 1	30	5 2 1		6 3 2		7 4 0	78	7 5 2
10	3 1 0	32	4 4 0	50	7 1 0		6 5 2	80	8 4 0
11	3 1 1	33	5 2 2		5 5 0	66	8 1 1	81	9 0 0
12	2 2 2		4 4 1		5 4 3		7 4 1		8 4 1
13	3 2 0	34	5 3 0	51	7 1 1		5 5 4		7 4 4
14	3 2 1		4 3 3		5 5 1	67	7 3 3		6 6 3
16	4 0 0	35	5 3 1	52	6 4 0	68	8 2 0	82	9 1 0
17	4 1 0	36	6 0 0	53	7 2 0		6 4 4		8 3 3
	3 2 2		4 4 2		6 4 1	69	8 2 1	83	9 1 1
18	4 1 1	37	6 1 0	54	7 2 1		7 4 2		7 5 3
	3 3 0	38	6 1 1		6 3 3	70	6 5 3	84	8 4 2
19	3 3 1		5 3 2		5 5 2	72	8 2 2	85	9 2 0
20	4 2 0	40	6 2 0	56	6 4 2		6 6 0		7 6 0
21	4 2 1	41	6 2 1	57	7 2 2	73	8 3 0	86	9 2 1
22	3 3 2		5 4 0		5 4 4		6 6 1		7 6 1
24	4 2 2		4 4 3	58	7 3 0				6 5 5