

2.6. SMALL-ANGLE TECHNIQUES

direction with a constant cross section of arbitrary shape (long cylinders, parallelepipeds, *etc.*) The cross section A (with maximum dimension d) should be small in comparison to the length of the whole particle L :

$$d \ll L \quad L = (D^2 - d^2)^{1/2} \simeq D. \quad (2.6.1.40)$$

The scattering curve of such a particle can be written as

$$I(h) = L(\pi/h)I_c(h), \quad (2.6.1.41)$$

where the function $I_c(h)$ is related only to the cross section and the factor $1/h$ is characteristic for rod-like particles (Kratky & Porod, 1948; Porod, 1982). The cross-section function $I_c(h)$ is

$$I_c(h) = (L\pi)^{-1}I(h)h = \text{constant} \times I(h)h. \quad (2.6.1.42)$$

This function was used in the previous subsection for the determination of the cross-section parameters R_c , A , and M_c . In addition, we have

$$I_c(h) = 2\pi \int_0^\infty p_c(r)J_0(hr) dr, \quad (2.6.1.43)$$

where $J_0(hr)$ is the zero-order Bessel function and

$$p_c(r) = \frac{1}{2\pi} \int_0^\infty I_c(h)(hr)J_0(hr) dh \quad (2.6.1.44)$$

(Glatter, 1982a). The function $p_c(r)$ is the PDDF of the cross section with

$$p_c(r) = r\gamma_c(r) = \langle \Delta\rho(\mathbf{r}_c) * \Delta\rho(-\mathbf{r}_c) \rangle. \quad (2.6.1.45)$$

The symbol $*$ stands for the mathematical operation called convolution and the symbol $\langle \rangle$ means averaging over all directions in the plane of the cross section. Rod-like particles with a constant cross section show a linear descent of $p(r)$ for $r \gg d$ if $D > 2.5d$. The slope of this linear part is proportional to the square of the area of the cross section,

$$\frac{dp}{dr} = -\frac{A^2\Delta\rho^2}{2}. \quad (2.6.1.46)$$

The PDDF's of parallelepipeds with the same cross section but different length L are shown in Fig. 2.6.1.6. The maximum corresponds to the cross section and the point of inflection r_i gives a rough indication for the size of the cross section. This is shown more clearly in Fig. 2.6.1.7, where three parallelepipeds with equal cross section area A but different cross-section dimensions are shown. If we find from the overall PDDF that the particle under investigation is a rod-like particle, we can use the

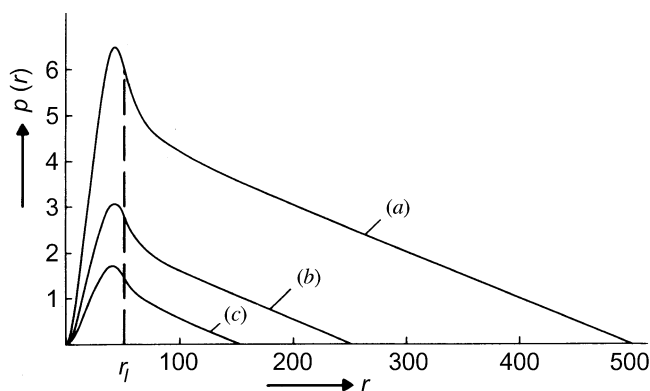


Fig. 2.6.1.6. Distance distributions from homogeneous parallelepipeds with edge lengths of: (a) $50 \times 50 \times 500 \text{ \AA}$; (b) $50 \times 50 \times 250 \text{ \AA}$; (c) $50 \times 50 \times 150 \text{ \AA}$.

PDDF of the cross section $p_c(r)$ to obtain more information on the cross section (Glatter, 1980a).

Flat particles. Flat particles, *i.e.* particles elongated in two dimensions (discs, flat parallelepipeds), with a constant thickness T much smaller than the overall dimensions D , can be treated in a similar way. The scattering function can be written as

$$I(h) = A \frac{2\pi}{h^2} I_t(h), \quad (2.6.1.47)$$

where $I_t(h)$ is the so-called thickness factor (Kratky & Porod, 1948) or

$$I_t(h) = (A2\pi)^{-1}I(h)h = \text{constant} \times I(h)h^2, \quad (2.6.1.48)$$

which can be used for the determination of R_t , T , and M_t . In addition, we have again:

$$I_t(h) = 2 \int_0^\infty p_t(r) \cos(hr) dr \quad (2.6.1.49)$$

and

$$\begin{aligned} p_t(r) &= \gamma_t(r) - \frac{1}{\pi} \int_0^\infty I_t(h) \cos(hr) dh \\ &= \Delta\rho_t(r) * \Delta\rho_t(-r). \end{aligned} \quad (2.6.1.50)$$

PDDF's from flat particles do not show clear features and therefore it is better to study $f(r) = p(r)/r$ (Glatter, 1979). The corresponding functions for lamellar particles with the same basal plane but different thickness are shown in Fig. 2.6.1.7(b). The marked transition points in Fig. 2.6.1.7(b) can be used to determine the thickness. The PDDF of the thickness $p_t(r)$ can give more information in such cases, especially for inhomogeneous particles (see below).

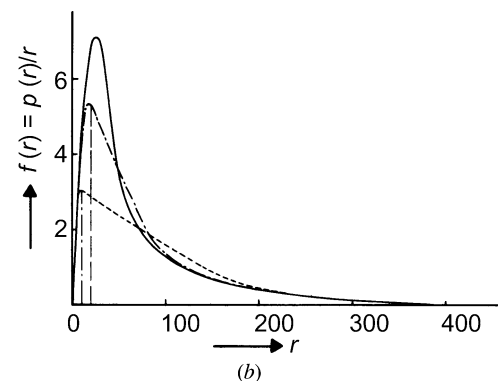
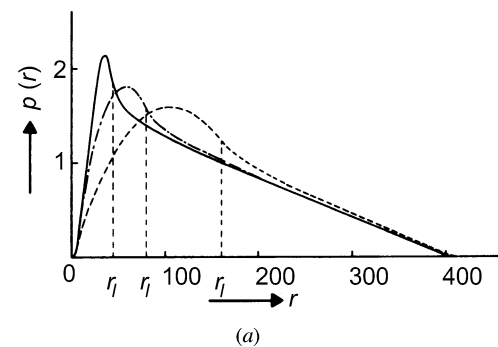


Fig. 2.6.1.7. Three parallelepipeds with constant length L (400 \AA) and a constant cross section but varying length of the edges: — $40 \times 40 \text{ \AA}$; — · — $80 \times 20 \text{ \AA}$; - - - $160 \times 10 \text{ \AA}$. (a) $p(r)$ function. (b) $f(r) = p(r)/r$.