Table 9.8.3.1(*b*). (2 + 2)-*Dimensional Bravais classes for incommensurate structures*

The holohedral point group K_s is given in terms of its external and internal parts, K_E and K_I, respectively. The basis of the vector module M^* contains two modulation wavevectors and the reflections are given by $h\mathbf{a}^* + k\mathbf{b}^* + m_1\mathbf{q}_1 + m_2\mathbf{q}_2$. If $\mathbf{q}_1^r$ or $\mathbf{q}_2^r$ are not zero, there are corresponding centring translations in four-dimensional space. The conventional basis $(\mathbf{a}_c^*, \mathbf{b}_c^*, \mathbf{q}_1^i, \mathbf{q}_2^i)$ for the vector module M^* is chosen such that $\mathbf{q}_1^r = \mathbf{q}_2^r = 0$. The basis vectors are indicated by their components with respect to the conventional basis $\mathbf{a}^*$, $\mathbf{b}^*$ of the lattice Λ^* of main reflections.

No.	Symbol	K_E	K_I	$\mathbf{q}_1$	$\mathbf{q}_2$	Conventional basis	Centring
Oblique							
1	$2p(\alpha\beta, \lambda\mu)$	2	2	$(\alpha\beta)$	$(\lambda\mu)$	(10), (01), $(\alpha\beta)$, $(\lambda\mu)$	
Rectangular							
2	$mmp(0\beta, 0\mu)$	mm	12	(0β)	(0μ)	(10), (01), (0β), (0μ)	
3	$mmp(\frac{1}{2}\beta, 0\mu)$	mm	12	$(\frac{1}{2}\beta)$	(0μ)	$(\frac{1}{2}0)$, (01), (0β), (0μ)	$\frac{1}{2}0\frac{1}{2}0$
4	$mmp(\alpha 0, 0\mu)$	mm	mm	$(\alpha 0)$	(0μ)	(10), (01), $(\alpha 0)$, (0μ)	
5	$mmp(\alpha\frac{1}{2}, 0\mu)$	mm	mm	$(\alpha\frac{1}{2})$	(0μ)	(10), $(0\frac{1}{2})$, $(\alpha 0)$, (0μ)	$0\frac{1}{2}\frac{1}{2}0$
6	$mmp(\alpha\frac{1}{2}, \frac{1}{2}\mu)$	mm	mm	$(\alpha\frac{1}{2})$	$(\frac{1}{2}\mu)$	$(\frac{1}{2}0)$, $(0\frac{1}{2})$, $(\alpha 0)$, (0μ)	$\frac{1}{2}00\frac{1}{2}$, $0\frac{1}{2}\frac{1}{2}0$
7	$mmp(\alpha\beta)$	mm	mm	$(\alpha\beta)$	$(\alpha\bar{\beta})$	(10), (01), $(\alpha 0)$, (0β)	$00\frac{1}{2}\frac{1}{2}$
8	$mmc(0\beta, 0\mu)$	mm	12	(0β)	(0μ)	(10), (01), (0β), (0μ)	$\frac{1}{2}\frac{1}{2}00$
9	$mmc(\alpha 0, 0\mu)$	mm	mm	$(\alpha 0)$	(0μ)	(10), (01), $(\alpha 0)$, (0μ)	$\frac{1}{2}\frac{1}{2}00$
10	$mmc(\alpha\beta)$	mm	mm	$(\alpha\beta)$	$(\alpha\bar{\beta})$	(10), (01), $(\alpha 0)$, (0β)	$\frac{1}{2}\frac{1}{2}00$, $00\frac{1}{2}\frac{1}{2}$
Square							
11	$4p(\alpha\beta)$	4	4	$(\alpha\beta)$	$(\bar{\beta}\alpha)$	(10), (01), $(\alpha\beta)$, $(\bar{\beta}\alpha)$	
12	$4mp(\alpha 0)$	$4m$	$4m$	$(\alpha 0)$	(0α)	(10), (01), $(\alpha 0)$, (0α)	
13	$4mp(\alpha\frac{1}{2})$	$4m$	$4m$	$(\alpha\frac{1}{2})$	$(\frac{1}{2}\alpha)$	$(\frac{1}{2}\frac{1}{2})$, $(\bar{\frac{1}{2}}\frac{1}{2})$, $(\gamma\gamma)$, $(\delta\bar{\delta})$ $\gamma = (2\alpha+1)/4$, $\delta = (2\alpha-1)/4$	$\frac{1}{2}\frac{1}{2}00$, $00\frac{1}{2}\frac{1}{2}$
14	$4mp(\alpha\alpha)$	$4\dot{m}$	$4\ddot{m}$	$(\alpha\alpha)$	$(\bar{\alpha}\alpha)$	(10), (01), $(\alpha\alpha)$, $(\bar{\alpha}\alpha)$	
Hexagonal							
15	$6p(\alpha\beta)$	6	6	$(\alpha\beta)$	$(\bar{\beta}\alpha+\beta)$	(10), (01), $(\alpha\beta)$, $(\bar{\beta}\alpha+\beta)$	
16	$6mp(\alpha 0)$	$6m$	$6m$	$(\alpha 0)$	(0α)	(10), (01), $(\alpha 0)$, (0α)	
17	$6mp(\alpha\alpha)$	$6\dot{m}$	$6\ddot{m}$	$(\alpha\alpha)$	$(\bar{\alpha}2\alpha)$	(10), (01), $(\alpha\alpha)$, $(\bar{\alpha}2\alpha)$	

mentioned above, the two-line symbol is $B_{1\ \bar{1}}^{2/mB}$. This symbol has the advantage that the internal transformation (the value of ε) is explicitly given for the corresponding generators. It has, however, certain typographical drawbacks. It is rare for the printer to put the symbol together in the correct manner: $B_{1\ \bar{1}}^{2/mB}$.

In Tables 9.8.3.1(*a*), (*b*) and 9.8.3.2(*a*), (*b*) the symbols for the $(2+d)$- and $(3+1)$-dimensional Bravais classes are given in the one-line form. It is, however, easy to derive from each one-line symbol the corresponding two-line symbol because the bottom line for the two-line symbol appears in the tables as the internal part of the point-group symbol.

The number of symbols in the bottom line of the two-line symbol should be equal to that of the generators given in the top line. A symbol '1' is used in the bottom line if the corresponding R_I is the unit transformation. If necessary, a mirror perpendicular to a crystal axis is indicated by $\dot{m}$ and one that is not by $\ddot{m}$. This situation only occurs for $d \geq 2$. So the $(2+2)$-dimensional class P_{4m}^{4mp} is actually $P_{4\dot{m}}^{4\dot{m}p}$ and is different from the class $P_{4\ddot{m}}^{4\dot{m}p}$. In a one-line symbol, their difference is apparent, the first being $4mp(\alpha 0)$, whereas the second is $4mp(\alpha\alpha)$.

9.8.3.2. *Table for geometric and arithmetic crystal classes*

In Table 9.8.3.3, the geometric and the arithmetic crystal classes of $(3+1)$-dimensional superspace are given.

The symbols for *geometric crystal classes* indicate the pairs $[R, \varepsilon(R)]$ of the generators of the point group. This is done by giving the crystal class for the point group K_E and the symbols for the corresponding elements of K_I. So, for example, the geometric crystal class belonging to the holohedral point group of the Bravais class $2/mB(0\frac{1}{2}\gamma)$, mentioned above, is $(2/m, 1\bar{1})$.

The notation for the *arithmetic crystal classes* is similar to that for the Bravais classes. In the tables, their one-line symbols are given. They consist of the (modified) symbol of the three-dimensional symmorphic space group and, in parentheses, the appropriate components of the modulation wavevector. The three arithmetic crystal classes implying a lattice belonging to the Bravais class $2/mB(0\frac{1}{2}\gamma)$ are $2B(0\frac{1}{2}\gamma)$, $mB(0\frac{1}{2}\gamma)$, and $2/mB(0\frac{1}{2}\gamma)$. The corresponding geometric crystal classes are $(2, 1)$, $(m, \bar{1})$, and $(2/m, 1\bar{1})$.

9.8.3.3. *Tables of superspace groups*

9.8.3.3.1. *Symmetry elements*

The transformations g_s belonging to a $(3+1)$-dimensional superspace group consist of a point-group transformation R_s given by the integral matrix $\Gamma(R)$ and of the associated translation. So the superspace group is determined by the arithmetic crystal class of its point group and the corresponding translational components. The symbol for the arithmetic crystal

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