Table 1.5.2.4. *List of the magnetic classes in which ferromagnetism is admitted*

(*a*) Triclinic

Symbol of symmetry class		Allowed direction of $\mathbf{M}_s$
Schoenflies	Hermann–Mauguin	
$\boldsymbol{C}_1$	1	Any
$\boldsymbol{C}_i$	$\bar{1}$	Any

(*b*) Monoclinic

Symbol of symmetry class		Allowed direction of $\mathbf{M}_s$
Schoenflies	Hermann–Mauguin	
$\boldsymbol{C}_2$	2	$\parallel 2$
$\boldsymbol{C}_2(\boldsymbol{C}_1)$	$2'$	$\perp 2'$
$\boldsymbol{C}_s = \boldsymbol{C}_{1h}$	m	$\perp m$
$\boldsymbol{C}_s(\boldsymbol{C}_1)$	m'	$\parallel m'$
$\boldsymbol{C}_{2h}$	$2/m$	$\parallel 2$
$\boldsymbol{C}_{2h}(\boldsymbol{C}_i)$	$2'/m'$	$\parallel m'$

(*c*) Orthorhombic

Symbol of symmetry class		Allowed direction of $\mathbf{M}_s$
Schoenflies	Hermann–Mauguin	
$\boldsymbol{D}_2(\boldsymbol{C}_2)$	$22'2'$	$\parallel 2$
$\boldsymbol{C}_{2v}(\boldsymbol{C}_2)$	$m'm'2$	$\parallel 2$
$\boldsymbol{C}_{2v}(\boldsymbol{C}_s)$	$m'm2'$	$\perp m$
$\boldsymbol{D}_{2h}(\boldsymbol{C}_{2h})$	$mm'm'$	$\perp m$

(*d*) Tetragonal

Symbol of symmetry class		Allowed direction of $\mathbf{M}_s$
Schoenflies	Hermann–Mauguin	
$\boldsymbol{C}_4$	4	$\parallel 4$
$\boldsymbol{S}_4$	$\bar{4}$	$\parallel \bar{4}$
$\boldsymbol{C}_{4h}$	$4/m$	$\parallel 4$
$\boldsymbol{D}_4(\boldsymbol{C}_4)$	$42'2'$	$\parallel 4$
$\boldsymbol{C}_{4v}(\boldsymbol{C}_4)$	$4m'm'$	$\parallel 4$
$\boldsymbol{D}_{2d}(\boldsymbol{S}_4)$	$\bar{4}2'm'$	$\parallel \bar{4}$
$\boldsymbol{D}_{4h}(\boldsymbol{C}_{4h})$	$4/mm'm'$	$\parallel 4$

(*e*) Trigonal

Symbol of symmetry class		Allowed direction of $\mathbf{M}_s$
Schoenflies	Hermann–Mauguin	
$\boldsymbol{C}_3$	3	$\parallel 3$
$\boldsymbol{S}_6$	$\bar{3}$	$\parallel 3$
$\boldsymbol{D}_3(\boldsymbol{C}_3)$	$32'$	$\parallel 3$
$\boldsymbol{C}_{3v}(\boldsymbol{C}_3)$	$3m'$	$\parallel 3$
$\boldsymbol{D}_{3d}(\boldsymbol{S}_6)$	$\bar{3}m'$	$\parallel 3$

(*f*) Hexagonal

Symbol of symmetry class		Allowed direction of $\mathbf{M}_s$
Schoenflies	Hermann–Mauguin	
$\boldsymbol{C}_6$	6	$\parallel 6$
$\boldsymbol{C}_{3h}$	$\bar{6}$	$\parallel \bar{6}$
$\boldsymbol{C}_{6h}$	$6/m$	$\parallel 6$
$\boldsymbol{D}_6(\boldsymbol{C}_6)$	$62'2'$	$\parallel 6$
$\boldsymbol{C}_{6v}(\boldsymbol{C}_6)$	$6m'm'$	$\parallel 6$
$\boldsymbol{D}_{3h}(\boldsymbol{C}_{3h})$	$\bar{6}m'2'$	$\parallel \bar{6}$
$\boldsymbol{D}_{6h}(\boldsymbol{C}_{6h})$	$6/mm'm'$	$\parallel 6$

the same for each member of a family. Each family begins with a trivial magnetic point group. It contains the same elements as the corresponding crystallographic point group; its Schoenflies symbol contains no brackets. For each nontrivial point group, the list of the elements of symmetry begins with the non-primed elements, which belong to a subgroup $\mathcal{H}$ of the head of the family $\mathcal{G}$. The number of the primed elements is equal to the number of non-primed ones and the total number of the elements is the same for all point groups of one family.

The overall number of the magnetic point groups of all three types is 122. There are two general statements concerning the magnetic point groups. The element $RC_3 = 3'$ does not appear in any of the magnetic point groups of type 3. Only trivial magnetic point groups (of both first and second type) belong to the families containing the point groups $\boldsymbol{C}_1 = 1$, $\boldsymbol{C}_3 = 3$ and $\boldsymbol{T} = 23$.

Only 31 magnetic point groups allow ferromagnetism. The different types of ferromagnetism (one-sublattice ferromagnet, ferrimagnet, weak ferromagnet, any magnetic order with nonzero magnetization) cannot be distinguished by their magnetic symmetry. Ferromagnetism is not admitted in any point group of type 1. For the magnetic point groups of the second type, ferromagnetism is not allowed if the point group contains more than one symmetry axis, more than one mirror plane or a mirror plane that is parallel to the axis. The same restrictions are valid for the point groups of type 3 (if the corresponding elements are not multiplied by R). If the point group contains $\bar{1}'$, ferromagnetic order is also forbidden. There are the following rules for the orientation of the axial vector of ferromagnetic magnetization $\mathbf{M}$: $\mathbf{M} \parallel N, \mathbf{M} \perp 2', \mathbf{M} \perp m, \mathbf{M} \parallel m'$. Table 1.5.2.4 lists those magnetic point groups that admit ferromagnetic order (Tavger, 1958). The allowed direction of the magnetization vector is given for every point group. Ferromagnetic order is allowed in 13 point groups of the second type and 18 point groups of the third type.

All 31 point groups of magnetic symmetry allowing ferromagnetism are subgroups of the infinite noncrystallographic group

$$\boldsymbol{D}_{\infty h}(\boldsymbol{C}_{\infty h}) = \frac{\infty}{m}\frac{2'}{m'}.$$

The transition from a paramagnetic to a ferromagnetic state is always accompanied by a change of the magnetic symmetry.

1.5.2.2. *Magnetic lattices*

If the point group of symmetry describes the macroscopic properties of a crystal, its microscopic structure is determined by the space group, which contains the group of translations $\mathcal{T}$ as a subgroup. The elements $\mathbf{t}$ of $\mathcal{T}$ are defined by the following relation:

$$\mathbf{t} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2 + n_3\mathbf{a}_3, \qquad (1.5.2.3)$$

where $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ are basic primitive translation vectors and n_1, n_2, n_3 are arbitrary integers. The set of points $\mathbf{r}'$ obtained by applying all the translations of the group $\mathcal{T}$ to any point $\mathbf{r}$ defines a lattice. All sites of the crystallographic lattice are equivalent.

The structure of the ordered magnetics is described by the magnetic lattices and corresponding magnetic translation groups $\mathcal{M}_T$. In the magnetic translation groups $\mathcal{M}_T$, some of the elements $\mathbf{t}$ may be multiplied by R (we shall call them primed translations). The magnetic lattices then have two types of sites, which are not equivalent. One set is obtained by non-primed translations and the other set by the primed ones. The magnetic translation group $\mathcal{M}_T$ is isometric to the crystallographic one $\mathcal{G}_0$ that is obtained by replacing R by E in $\mathcal{M}_T$.

There are trivial magnetic translation groups, in which none of the translation elements is multiplied by R. The magnetic lattices of these groups coincide with crystallographic lattices.

Nontrivial magnetic translation groups can be constructed in analogy to relation (1.5.2.2). Zamorzaev (1957) showed that every translation group $\mathcal{T}$ has seven subgroups of index 2. If the basic primitive translations of the group $\mathcal{T}$ are $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, then the basic primitive translations of the seven subgroups $\mathcal{H}$ can be chosen as follows (see also Opechowski & Guccione, 1965)

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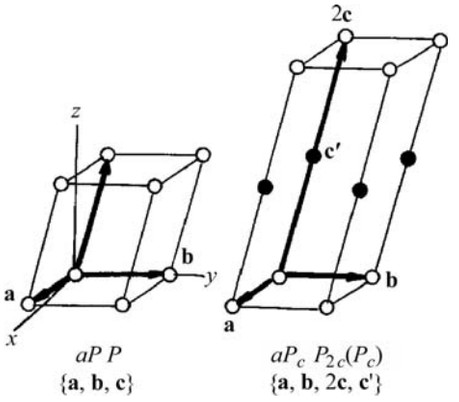

Fig. 1.5.2.1. Magnetic lattices of the triclinic system.

$$\mathcal{H}_1 : 2\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3 \quad (1.5.2.4)$$
$$\mathcal{H}_2 : \mathbf{a}_1, 2\mathbf{a}_2, \mathbf{a}_3 \quad (1.5.2.5)$$
$$\mathcal{H}_3 : \mathbf{a}_1, \mathbf{a}_2, 2\mathbf{a}_3 \quad (1.5.2.6)$$
$$\mathcal{H}_4 : 2\mathbf{a}_1, \mathbf{a}_1 + \mathbf{a}_2, \mathbf{a}_3 \quad (1.5.2.7)$$
$$\mathcal{H}_5 : 2\mathbf{a}_2, \mathbf{a}_2 + \mathbf{a}_3, \mathbf{a}_1 \quad (1.5.2.8)$$
$$\mathcal{H}_6 : 2\mathbf{a}_3, \mathbf{a}_3 + \mathbf{a}_1, \mathbf{a}_2 \quad (1.5.2.9)$$
$$\mathcal{H}_7 : 2\mathbf{a}_1, \mathbf{a}_1 + \mathbf{a}_2, \mathbf{a}_1 + \mathbf{a}_3. \quad (1.5.2.10)$$

As an example, let us consider the case (1.5.2.5). In this case, the subgroup $\mathcal{H}$ consists of the following translations:

$$\mathbf{t}(\mathcal{H}) = n_1\mathbf{a}_1 + 2n_2\mathbf{a}_2 + n_3\mathbf{a}_3. \quad (1.5.2.11)$$

Therefore the elements G_i of $(\mathcal{T} - \mathcal{H})$ [which corresponds to $(\mathcal{P} - \mathcal{H})$ in relation (1.5.2.2)] must have the following form:

$$\mathbf{t}(G_i) = n_1\mathbf{a}_1 + (2n_2 + 1)\mathbf{a}_2 + n_3\mathbf{a}_3. \quad (1.5.2.12)$$

The corresponding magnetic translation group consists of the elements (1.5.2.12) multiplied by R and the elements (1.5.2.11).

The crystallographic lattices are classified into Bravais types or Bravais lattices. The magnetic lattices are classified into Bravais types of magnetic lattices. It turns out that there are 22 nontrivial magnetic Bravais types. Together with the trivial ones, there are 36 magnetic Bravais lattices.

Two types of smallest translation-invariant cells are in common use for the description of magnetically ordered structures: the crystallographic cell obtained if the magnetic order is neglected and the magnetic cell, which takes the magnetic order into account. The list of the basic translations of all the magnetic Bravais lattices was given by Zamorzaev (1957). The diagrams of the magnetic unit cells were obtained by Belov *et al.* (1957).

In Figs. 1.5.2.1–1.5.2.7, the diagrams of the magnetic unit cells of all 36 Bravais types are sketched in such a way that it is clear to which family the given cell belongs. All the cells of one family are displayed in one row. Such a row begins with the cell of the trivial magnetic lattice. All nontrivial cells of a family change into the trivial one of this family if R is replaced by E (to draw these diagrams we used those published by Opechowski & Guccione, 1965). Open and full circles are used to show the primed and unprimed translations. A line connecting two circles of the same type is an unprimed translation; a line connecting two circles of different types is a primed translation. The arrows in the trivial magnetic cell represent the primitive (primed or unprimed) translations for all the magnetic lattices of the family. The arrows in the nontrivial cells are primitive translations of the magnetic unit cell. The magnetic unit cell of a nontrivial magnetic lattice is generated by unprimed translations only. Its volume is twice the volume of the smallest cell generated by all (primed and unprimed) translations. The reason for this is that one of the primitive translations of the magnetic cell is twice a primitive primed translation. The crystallographic cell of many simple collinear or weakly non-collinear structures coincides with the smallest cell generated by the primed and unprimed translations. However, there are also magnetic structures with more complicated transformations from the crystallographic to the magnetic unit cell. The second line after each part of Figs. 1.5.2.1–1.5.2.7 gives, between braces, an extended vector basis of the magnetic translation group (Shubnikov & Koptsik, 1972). The first line

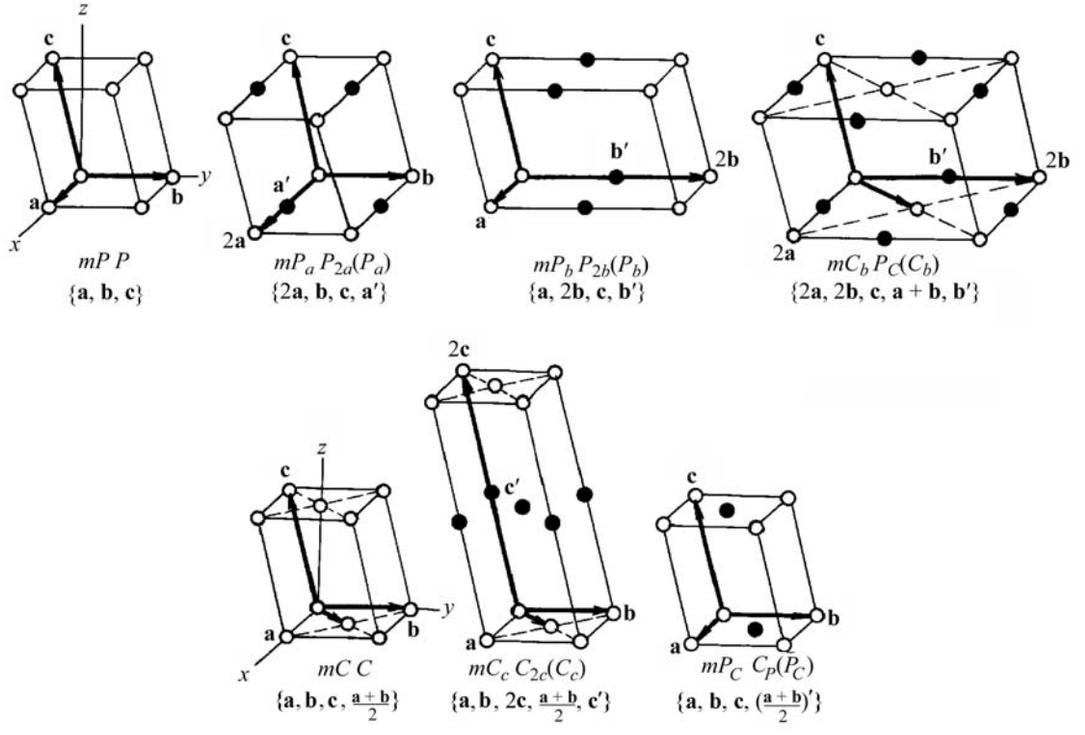

Fig. 1.5.2.2. Magnetic lattices of the monoclinic system (the y axis is the twofold axis).

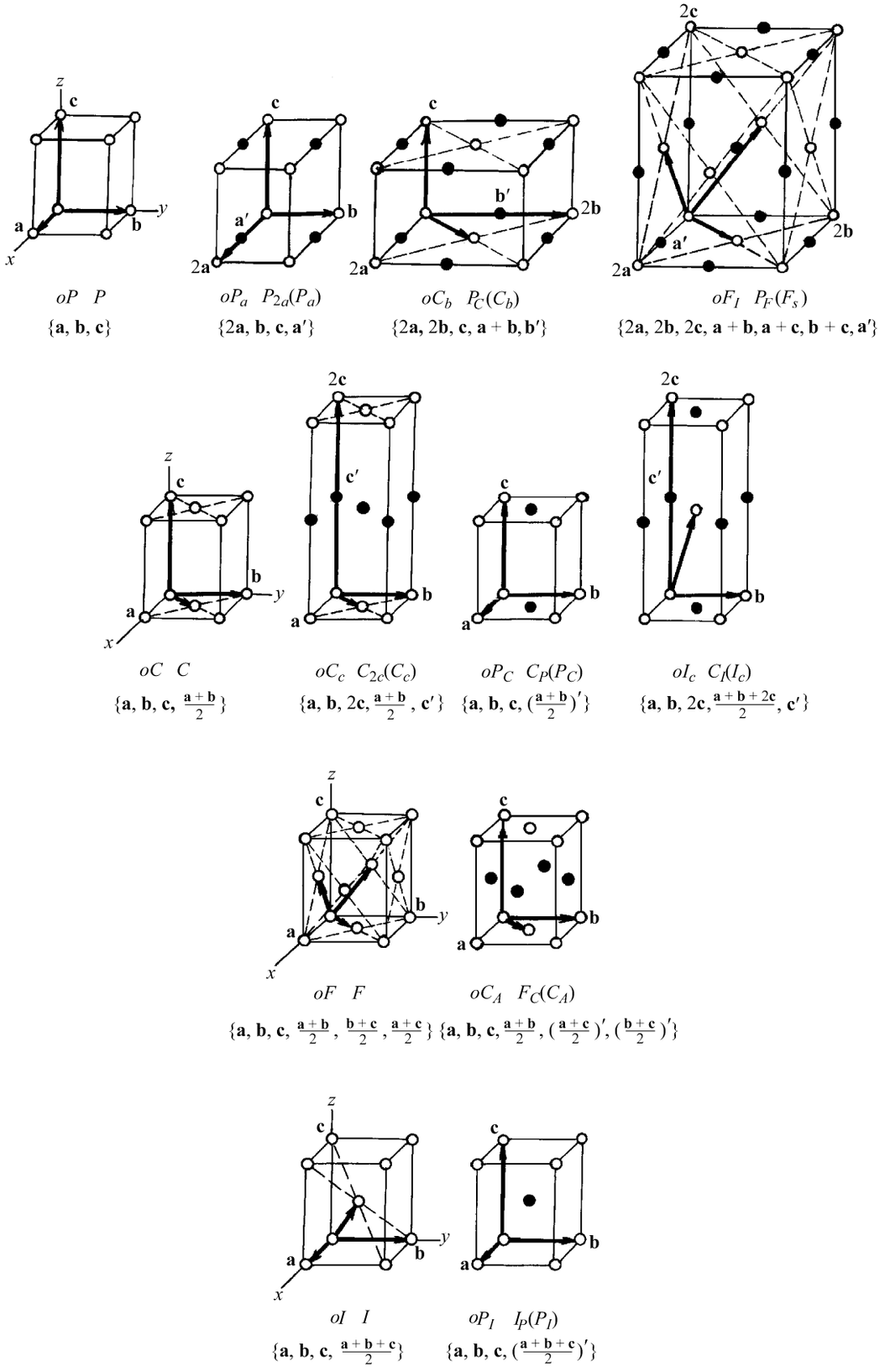

Fig. 1.5.2.3. Magnetic lattices of the orthorhombic system.

gives two symbols for each Bravais type: the symbol to the right was introduced by Opechowski & Guccione (1965). The symbol to the left starts with a lower-case letter giving the crystal system followed by a capital letter giving the centring type of the cell defined by the unprimed translations (*P*: primitive; *C*, *A*, *B*: *C*-, *A*-, *B*-centred; *I*: body-centred; *F*: all-face-centred). The subscript, which appears for the nontrivial Bravais types, indicates the translations that are multiplied by time inversion *R*.

Ferromagnetism is allowed only in trivial magnetic Bravais lattices. All nontrivial magnetic lattices represent antiferromagnetic order. There are only two magnetic sublattices in the simplest antiferromagnetic structures; one sublattice consists

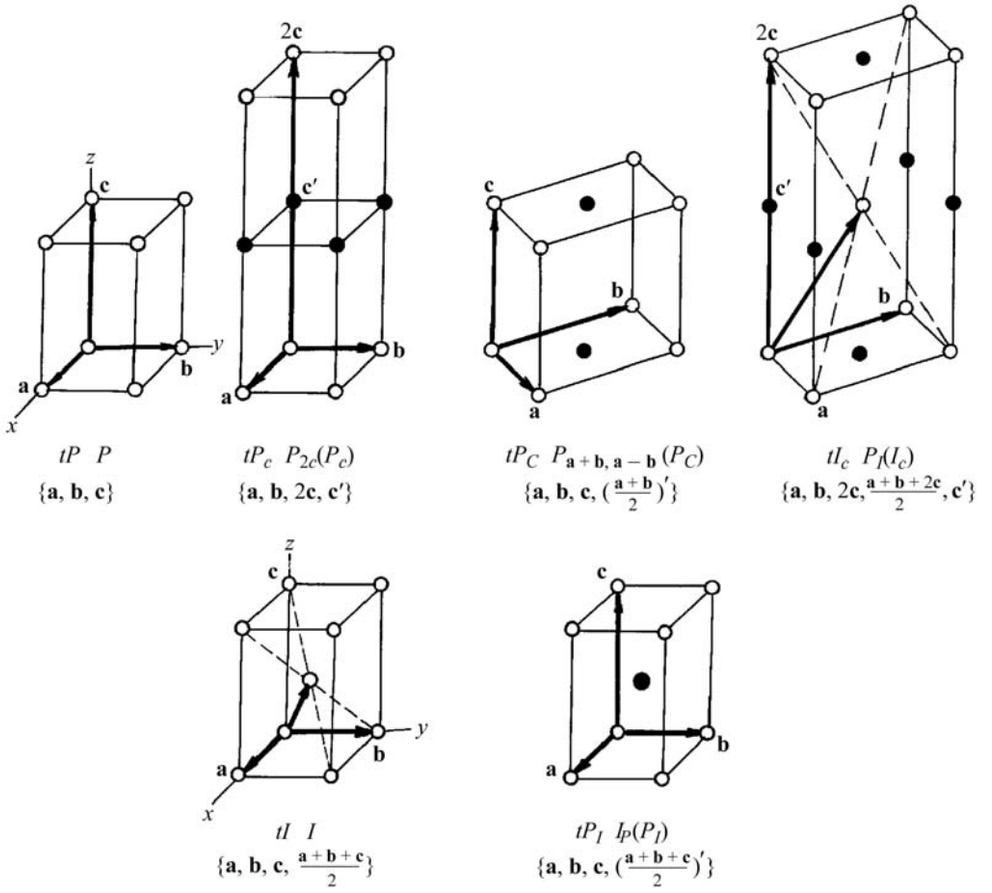

Fig. 1.5.2.4. Magnetic lattices of the tetragonal system.

of the magnetic ions located in the black sites and the other of the ions located in the white sites. All the magnetic moments of one sublattice are oriented in one direction and those of the other sublattice in the opposite direction. However, antiferromagnetism is allowed also in trivial lattices if the (trivial) magnetic cell contains more than one magnetic ion. The magnetic point group must be nontrivial in this case. The situation is more complicated in case of strongly non-collinear structures. In such structures (triangle, 90° *etc.*), the magnetic lattice can differ from the crystallographic one despite the fact that none of the translations is multiplied by *R*. The magnetic elementary cell will possess three or four magnetic ions although the crystallographic cell possesses only one. An example of such a situation is shown in Fig. 1.5.1.3(*c*). More complicated structures in which the magnetic lattice is incommensurate with the crystallographic one also exist. We shall not discuss the problems of such systems in this chapter.

1.5.2.3. *Magnetic space groups*

There are 1651 magnetic space groups $\mathcal{M}_G$, which can be divided into three types. Type I, $\mathcal{M}_{G1}$, consists of the 230 crystallographic space groups to which *R* is added. Crystals belonging to these trivial magnetic space groups show no magnetic order; they are para- or diamagnetic.

Type II, $\mathcal{M}_{G2}$, consists of the same 230 crystallographic groups which do not include *R* in any form. In the ordered magnetics, which belong to the magnetic space groups of this type, the

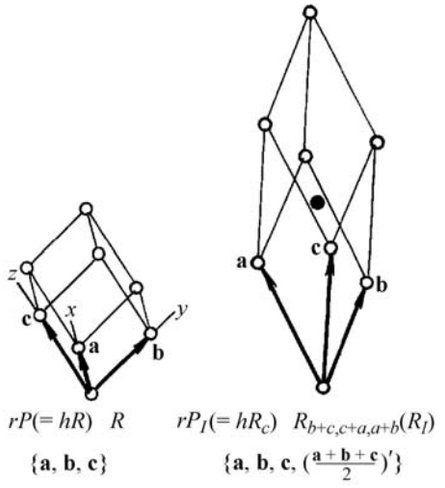

Fig. 1.5.2.5. Magnetic lattices of the rhombohedral system.

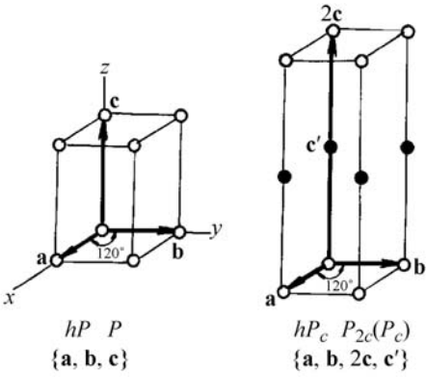

Fig. 1.5.2.6. Magnetic lattices of the hexagonal system.

references