No. 154 $P3_221$ D_3^6

$$\mathcal{G} = P3_221$$

Orientation orbit $(hkil)$	Conventional basis of the scanning group $\mathbf{a}'$	$\mathbf{b}'$	$\mathbf{d}$	Scanning group $\mathcal{H}$	Linear orbit $s\mathbf{d}$	Sectional layer group $\mathcal{L}(s\mathbf{d})$	
(0001)	$\mathbf{a}$	$\mathbf{b}$	$\mathbf{c}$	$P3_221$	$[0\mathbf{d},\ [\frac{1}{2}\mathbf{d},$ $\frac{1}{3}\mathbf{d},\ \|\ \frac{5}{6}\mathbf{d},$ $\frac{2}{3}\mathbf{d}]\quad \frac{1}{6}\mathbf{d}]$ $[\pm s\mathbf{d},\ (\pm s+\frac{1}{3})\mathbf{d},$ $(\pm s+\frac{2}{3})\mathbf{d}]$	$\widehat{c}_3211$ $\widehat{c}_2211$ $\widehat{c}_1211$ $p1$	L10 L10 L10 L01
$(01\overline{1}0)$	$\mathbf{c}$	$\mathbf{a}$	$\mathbf{a}+2\mathbf{b}$	$A121\ (\mathbf{a}'/6)$	$[0\mathbf{d},\ \frac{1}{2}\mathbf{d}]$ $[\frac{1}{4}\mathbf{d},\ \frac{3}{4}\mathbf{d}]$ $[\pm s\mathbf{d},\ (\pm s+\frac{1}{2})\mathbf{d}]$	$p121\ (\mathbf{a}'/6)$ $p12_11\ (\mathbf{a}'/6)$ $p1$	L08 L09 L01
$(\overline{1}010)$	$\mathbf{c}$	$\mathbf{b}$	$-(2\mathbf{a}+\mathbf{b})$	$A121\ (\mathbf{a}'/3)$	$[0\mathbf{d},\ \frac{1}{2}\mathbf{d}]$ $[\frac{1}{4}\mathbf{d},\ \frac{3}{4}\mathbf{d}]$ $[\pm s\mathbf{d},\ (\pm s+\frac{1}{2})\mathbf{d}]$	$p121\ (\mathbf{a}'/3)$ $p12_11\ (\mathbf{a}'/3)$ $p1$	L08 L09 L01
$(1\overline{1}00)$	$\mathbf{c}$	$-(\mathbf{a}+\mathbf{b})$	$(\mathbf{a}-\mathbf{b})$	$A121$	$[0\mathbf{d},\ \frac{1}{2}\mathbf{d}]$ $[\frac{1}{4}\mathbf{d},\ \frac{3}{4}\mathbf{d}]$ $[\pm s\mathbf{d},\ (\pm s+\frac{1}{2})\mathbf{d}]$	$p121$ $p12_11$ $p1$	L08 L09 L01
$(\overline{1}2\overline{1}0)$	$\mathbf{c}$	$2\mathbf{a}+\mathbf{b}$	$\mathbf{b}$	$A112\ (\mathbf{a}'/3)$	$[s\mathbf{d},\ (s+\frac{1}{2})\mathbf{d}]$	$p112\ (\mathbf{a}'/3)$	L03
$(\overline{1}\overline{1}20)$	$\mathbf{c}$	$(\mathbf{b}-\mathbf{a})$	$-(\mathbf{a}+\mathbf{b})$	$A112$	$[s\mathbf{d},\ (s+\frac{1}{2})\mathbf{d}]$	$p112$	L03
$(2\overline{1}\overline{1}0)$	$\mathbf{c}$	$-(\mathbf{a}+2\mathbf{b})$	$\mathbf{a}$	$A112\ (\mathbf{a}'/6)$	$[s\mathbf{d},\ (s+\frac{1}{2})\mathbf{d}]$	$p112\ (\mathbf{a}'/6)$	L03

No. 155 $R32$ D_3^7

$$\mathcal{G} = R32$$

Orientation orbit		Conventional basis of the scanning group			Scanning group	Linear orbit	Sectional layer group	
HEXAG. AXES $(hkil)$	RHOMB. AXES (hkl)	$\mathbf{a}'$	$\mathbf{b}'$	$\mathbf{d}$	$\mathcal{H}$	$s\mathbf{d}$	$\mathcal{L}(s\mathbf{d})$	
(0001)	(111)	$\mathbf{a}$	$\mathbf{b}$	$\mathbf{c}$	$R32$	$[0\mathbf{d},\ [\frac{1}{2}\mathbf{d},$ $\frac{1}{3}\mathbf{d},\ \|\ \frac{5}{6}\mathbf{d},$ $\frac{2}{3}\mathbf{d}]\quad \frac{1}{6}\mathbf{d}]$ $[\pm s\mathbf{d},\ (\pm s+\frac{1}{3})\mathbf{d},\ (\pm s+\frac{2}{3})\mathbf{d}]$	$p321$ $p321\ [(2\mathbf{a}+\mathbf{b})/3]$ $p321\ [(\mathbf{a}+2\mathbf{b})/3]$ $p3$	L68 L68 L68 L65
$(01\overline{1}0)$	$(11\overline{1})$	$\mathbf{c}$	$\mathbf{a}$	$-\mathbf{c}_r$	$I121$	$[0\mathbf{d},\ \frac{1}{2}\mathbf{d}]$	$p121$	L08
$(\overline{1}010)$	$(\overline{1}11)$	$\mathbf{c}$	$\mathbf{b}$	$-\mathbf{a}_r$		$[\frac{1}{4}\mathbf{d},\ \frac{3}{4}\mathbf{d}]$	$p12_11\ (\mathbf{a}'/4)$	L09
$(1\overline{1}00)$	$(1\overline{1}1)$	$\mathbf{c}$	$-(\mathbf{a}+\mathbf{b})$	$-\mathbf{b}_r$		$[\pm s\mathbf{d},\ (\pm s+\frac{1}{2})\mathbf{d}]$	$p1$	L01
$(\overline{1}2\overline{1}0)$	$(01\overline{1})$	$\mathbf{c}$	$\mathbf{a}_r$	$\mathbf{b}$	$I112$	$[s\mathbf{d},\ (s+\frac{1}{2})\mathbf{d}]$	$p112$	L03
$(\overline{1}\overline{1}20)$	$(\overline{1}01)$	$\mathbf{c}$	$\mathbf{b}_r$	$-(\mathbf{a}+\mathbf{b})$				
$(2\overline{1}\overline{1}0)$	$(1\overline{1}0)$	$\mathbf{c}$	$\mathbf{c}_r$	$\mathbf{a}$				

Arithmetic classes $321P$, $3m1P$ and $\overline{3}m1P$

Orientation orbit $(hkil)$	Conventional basis of the scanning group $\mathbf{a}'$	$\mathbf{b}'$	$\mathbf{d}$	Auxiliary basis of the scanning group $\widehat{\mathbf{a}}$	$\widehat{\mathbf{b}}$	$\widehat{\mathbf{c}}$
$(0h\overline{h}l)$	$\mathbf{a}$	$n(\mathbf{a}+2\mathbf{b})-m\mathbf{c}$	$p(\mathbf{a}+2\mathbf{b})+q\mathbf{c}$	$\mathbf{a}+2\mathbf{b}$	$\mathbf{c}$	$\mathbf{a}$
$(\overline{h}0hl)$	$\mathbf{b}$	$-n(2\mathbf{a}+\mathbf{b})-m\mathbf{c}$	$-p(2\mathbf{a}+\mathbf{b})+q\mathbf{c}$	$-(2\mathbf{a}+\mathbf{b})$	$\mathbf{c}$	$\mathbf{b}$
$(h\overline{h}0l)$	$-(\mathbf{a}+\mathbf{b})$	$n(\mathbf{a}-\mathbf{b})-m\mathbf{c}$	$p(\mathbf{a}-\mathbf{b})+q\mathbf{c}$	$\mathbf{a}-\mathbf{b}$	$\mathbf{c}$	$-(\mathbf{a}+\mathbf{b})$
l odd $\Rightarrow n=l, m=2h$; l even $\Rightarrow n=l/2, m=h$						

Arithmetic classes $321P$, $3m1P$ and $\overline{3}m1P$

Serial No.	150	152	154	156	158	164	165
Group type	D_3^2	D_3^4	D_3^6	C_{3v}^1	C_{3v}^2	D_{3d}^3	D_{3d}^4
Group	$P321$	$P3_121$	$P3_221$	$P3m1$	$P3c1$	$P\overline{3}m1$	$P\overline{3}c1$
$(0h\overline{h}l)$	$B112$	$B112$ $(\mathbf{c}/3)$	$B112$ $(\mathbf{c}/6)$	$B11m$	$B11b$	$B112/m$	$B112/b$
$(\overline{h}0hl)$		$B112$ $(\mathbf{c}/6)$	$B112$ $(\mathbf{c}/3)$				
$(h\overline{h}0l)$		$B112$	$B112$				

Centring type *R*

Arithmetic classes $32R$, $3mR$ and $\overline{3}mR$

Orientation orbit		Conventional basis of the scanning group			Auxiliary basis of the scanning group		
HEXAG. AXES $(hkil)$	RHOMB. AXES (hkl)	$\mathbf{a}'$	$\mathbf{b}'$	$\mathbf{d}$	$\widehat{\mathbf{a}}$	$\widehat{\mathbf{b}}$	$\widehat{\mathbf{c}}$
$(0h\overline{h}l)$	(hhl)	$\mathbf{a}$	$n\mathbf{c}-m\mathbf{c}_r$	$p\mathbf{c}+q\mathbf{c}_r$	$\mathbf{c}$	$\mathbf{c}_r$	$\mathbf{a}$
$(\overline{h}0hl)$	(lhh)	$\mathbf{b}$	$n\mathbf{c}-m\mathbf{a}_r$	$p\mathbf{c}+q\mathbf{a}_r$	$\mathbf{c}$	$\mathbf{a}_r$	$\mathbf{b}$
$(h\overline{h}0l)$	(hlh)	$-(\mathbf{a}+\mathbf{b})$	$n\mathbf{c}-m\mathbf{b}_r$	$p\mathbf{c}+q\mathbf{b}_r$	$\mathbf{c}$	$\mathbf{b}_r$	$-(\mathbf{a}+\mathbf{b})$
Transformation of indices from hexagonal to auxiliary monoclinic basis l odd $\Rightarrow n=l-2h, m=6h$; l even $\Rightarrow n=l/2-h, m=3h$							
Transformation of indices from rhombohedral to auxiliary monoclinic basis l odd $\Rightarrow n=l, m=2h+l$; l even $\Rightarrow n=l/2, m=h+l/2$							

Arithmetic classes $32R$, $3mR$ and $\overline{3}mR$

Serial No.		155	160	161	166	167
Group type		D_3^7	C_{3v}^5	C_{3v}^6	D_{3d}^5	D_{3d}^6
Group		$R32$	$R3m$	$R3c$	$R\overline{3}m$	$R\overline{3}c$
HEXAG. AXES	RHOMB. AXES					
$(0h\overline{h}l)$	(hhl)	$I112$	$I11m$	$I11a$	$I112/m$	$I112/a$
$(\overline{h}0hl)$	(lhh)					
$(h\overline{h}0l)$	(hlh)					