No. 165 $P\bar{3}c1$ D^4_{3d}

$$\mathcal{G} = P\bar{3}\tfrac{2}{c}1$$

Orientation orbit $(hkil)$	Conventional basis of the scanning group $\mathbf{a}'$	$\mathbf{b}'$	$\mathbf{d}$	Scanning group $\mathcal{H}$	Linear orbit $s\mathbf{d}$	Sectional layer group $\mathcal{L}(s\mathbf{d})$	
(0001)	$\mathbf{a}$	$\mathbf{b}$	$\mathbf{c}$	$P\bar{3}c1$	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$p\bar{3}$	L66
					$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$p321$	L68
					$[\pm s\mathbf{d}, (\pm s+\frac{1}{2})\mathbf{d}]$	$p3$	L65
$(01\bar{1}0)$	$\mathbf{c}$	$\mathbf{a}$	$\mathbf{a}+2\mathbf{b}$	$A12/a1$	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$p12/a1$	L16
$(\bar{1}010)$	$\mathbf{c}$	$\mathbf{b}$	$-(2\mathbf{a}+\mathbf{b})$		$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$p12_1/a1$ $(\mathbf{b}'/4)$	L17
$(1\bar{1}00)$	$\mathbf{c}$	$-(\mathbf{a}+\mathbf{b})$	$(\mathbf{a}-\mathbf{b})$		$[\pm s\mathbf{d}, (\pm s+\frac{1}{2})\mathbf{d}]$	$p1a1$	L12
$(\bar{1}2\bar{1}0)$	$\mathbf{c}$	$2\mathbf{a}+\mathbf{b}$	$\mathbf{b}$	$A112/a$	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$p112/a$	L07
$(\bar{1}\bar{1}20)$	$\mathbf{c}$	$(\mathbf{b}-\mathbf{a})$	$-(\mathbf{a}+\mathbf{b})$		$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$p112/n$ $(\mathbf{b}'/4)$	L07
$(2\bar{1}\bar{1}0)$	$\mathbf{c}$	$-(\mathbf{a}+2\mathbf{b})$	$\mathbf{a}$		$[\pm s\mathbf{d}, (\pm s+\frac{1}{2})\mathbf{d}]$	$p112$ $(\mathbf{a}'/4)$	L03

No. 166 $R\bar{3}m$ D^5_{3d}

$$\mathcal{G} = R\bar{3}\tfrac{2}{m}$$

Orientation orbit		Conventional basis of the scanning group			Scanning group	Linear orbit	Sectional layer group	
HEXAG. AXES $(hkil)$	RHOMB. AXES (hkl)	$\mathbf{a}'$	$\mathbf{b}'$	$\mathbf{d}$	$\mathcal{H}$	$s\mathbf{d}$	$\mathcal{L}(s\mathbf{d})$	
(0001)	(111)	$\mathbf{a}$	$\mathbf{b}$	$\mathbf{c}$	$R\bar{3}m$	$[0\mathbf{d},\ [\frac{1}{2}\mathbf{d},$	$p\bar{3}m1$	L72
						$\frac{1}{3}\mathbf{d},\ \|\ \frac{5}{6}\mathbf{d},$	$p\bar{3}m1$ $[(2\mathbf{a}+\mathbf{b})/3]$	L72
						$\frac{2}{3}\mathbf{d}]\quad \frac{1}{6}\mathbf{d}]$	$p\bar{3}m1$ $[(\mathbf{a}+2\mathbf{b})/3]$	L72
						$[\pm s\mathbf{d}, (\pm s+\frac{1}{3})\mathbf{d},$		
						$(\pm s+\frac{2}{3})\mathbf{d}]$	$p3m1$	L69
$(01\bar{1}0)$	$(11\bar{1})$	$\mathbf{c}$	$\mathbf{a}$	$-\mathbf{c}_r$	$I12/m1$	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$p12/m1$	L14
$(\bar{1}010)$	$(\bar{1}11)$	$\mathbf{c}$	$\mathbf{b}$	$-\mathbf{a}_r$		$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$p12_1/m1$ $[(\mathbf{a}'+\mathbf{b}')/4]$	L15
$(1\bar{1}00)$	$(1\bar{1}1)$	$\mathbf{c}$	$-(\mathbf{a}+\mathbf{b})$	$-\mathbf{b}_r$		$[\pm s\mathbf{d}, (\pm s+\frac{1}{2})\mathbf{d}]$	$p1m1$	L11
$(\bar{1}2\bar{1}0)$	$(01\bar{1})$	$\mathbf{c}$	$\mathbf{a}_r$	$\mathbf{b}$	$I112/m$	$[0\mathbf{d}, \frac{1}{2}\mathbf{d}]$	$p112/m$	L06
$(\bar{1}\bar{1}20)$	$(\bar{1}01)$	$\mathbf{c}$	$\mathbf{b}_r$	$-(\mathbf{a}+\mathbf{b})$		$[\frac{1}{4}\mathbf{d}, \frac{3}{4}\mathbf{d}]$	$p112/n$ $[(\mathbf{a}'+\mathbf{b}')/4]$	L07
$(2\bar{1}\bar{1}0)$	$(1\bar{1}0)$	$\mathbf{c}$	$\mathbf{c}_r$	$\mathbf{a}$		$[\pm s\mathbf{d}, (\pm s+\frac{1}{2})\mathbf{d}]$	$p112$	L03

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Arithmetic classes $321P$, $3m1P$ and $\overline{3}m1P$

Orientation orbit $(hkil)$	Conventional basis of the scanning group $\mathbf{a}'$	$\mathbf{b}'$	$\mathbf{d}$	Auxiliary basis of the scanning group $\widehat{\mathbf{a}}$	$\widehat{\mathbf{b}}$	$\widehat{\mathbf{c}}$
$(0h\overline{h}l)$	$\mathbf{a}$	$n(\mathbf{a}+2\mathbf{b})-m\mathbf{c}$	$p(\mathbf{a}+2\mathbf{b})+q\mathbf{c}$	$\mathbf{a}+2\mathbf{b}$	$\mathbf{c}$	$\mathbf{a}$
$(\overline{h}0hl)$	$\mathbf{b}$	$-n(2\mathbf{a}+\mathbf{b})-m\mathbf{c}$	$-p(2\mathbf{a}+\mathbf{b})+q\mathbf{c}$	$-(2\mathbf{a}+\mathbf{b})$	$\mathbf{c}$	$\mathbf{b}$
$(h\overline{h}0l)$	$-(\mathbf{a}+\mathbf{b})$	$n(\mathbf{a}-\mathbf{b})-m\mathbf{c}$	$p(\mathbf{a}-\mathbf{b})+q\mathbf{c}$	$\mathbf{a}-\mathbf{b}$	$\mathbf{c}$	$-(\mathbf{a}+\mathbf{b})$
l odd $\Rightarrow n=l, m=2h$; l even $\Rightarrow n=l/2, m=h$						

Arithmetic classes $321P$, $3m1P$ and $\overline{3}m1P$

Serial No.	150	152	154	156	158	164	165
Group type	D_3^2	D_3^4	D_3^6	C_{3v}^1	C_{3v}^2	D_{3d}^3	D_{3d}^4
Group	$P321$	$P3_121$	$P3_221$	$P3m1$	$P3c1$	$P\overline{3}m1$	$P\overline{3}c1$
$(0h\overline{h}l)$	$B112$	$B112$ $(\mathbf{c}/3)$	$B112$ $(\mathbf{c}/6)$	$B11m$	$B11b$	$B112/m$	$B112/b$
$(\overline{h}0hl)$		$B112$ $(\mathbf{c}/6)$	$B112$ $(\mathbf{c}/3)$				
$(h\overline{h}0l)$		$B112$	$B112$				

Centring type *R*

Arithmetic classes $32R$, $3mR$ and $\overline{3}mR$

Orientation orbit		Conventional basis of the scanning group			Auxiliary basis of the scanning group		
HEXAG. AXES $(hkil)$	RHOMB. AXES (hkl)	$\mathbf{a}'$	$\mathbf{b}'$	$\mathbf{d}$	$\widehat{\mathbf{a}}$	$\widehat{\mathbf{b}}$	$\widehat{\mathbf{c}}$
$(0h\overline{h}l)$	(hhl)	$\mathbf{a}$	$n\mathbf{c}-m\mathbf{c}_r$	$p\mathbf{c}+q\mathbf{c}_r$	$\mathbf{c}$	$\mathbf{c}_r$	$\mathbf{a}$
$(\overline{h}0hl)$	(lhh)	$\mathbf{b}$	$n\mathbf{c}-m\mathbf{a}_r$	$p\mathbf{c}+q\mathbf{a}_r$	$\mathbf{c}$	$\mathbf{a}_r$	$\mathbf{b}$
$(h\overline{h}0l)$	(hlh)	$-(\mathbf{a}+\mathbf{b})$	$n\mathbf{c}-m\mathbf{b}_r$	$p\mathbf{c}+q\mathbf{b}_r$	$\mathbf{c}$	$\mathbf{b}_r$	$-(\mathbf{a}+\mathbf{b})$
Transformation of indices from hexagonal to auxiliary monoclinic basis l odd $\Rightarrow n=l-2h, m=6h$; l even $\Rightarrow n=l/2-h, m=3h$							
Transformation of indices from rhombohedral to auxiliary monoclinic basis l odd $\Rightarrow n=l, m=2h+l$; l even $\Rightarrow n=l/2, m=h+l/2$							

Arithmetic classes $32R$, $3mR$ and $\overline{3}mR$

Serial No.		155	160	161	166	167
Group type		D_3^7	C_{3v}^5	C_{3v}^6	D_{3d}^5	D_{3d}^6
Group		$R32$	$R3m$	$R3c$	$R\overline{3}m$	$R\overline{3}c$
HEXAG. AXES	RHOMB. AXES					
$(0h\overline{h}l)$	(hhl)	$I112$	$I11m$	$I11a$	$I112/m$	$I112/a$
$(\overline{h}0hl)$	(lhh)					
$(h\overline{h}0l)$	(hlh)					