Subject index

Abelian group, 10, 31
Abstract group, 10
Action of a group on a set, 32
Additional centring translations, 43
Affine group, 18, 30
Affine mapping, 6, 30
Affine normalizer, 18, 24
Affine space, 30
Affine space-group type, 14
Alternating group, 34
Aristotype, 5, 25
Associative law, 10
Augmented matrix, 8
Automorphism, 33
Automorphism group, 30
Axis of a rotation, 6
Axis of a rotoinversion, 7
Axis of a screw rotation, 6

Bärnighausen tree, 5, 54
Basic structure, 5, 25
Basis, 7
  conventional, 7, 13
  crystallographic, 7
  lattice, 7
  primitive, 7
Basis of a vector space, 29
Basis transformation, 46–47, 51
Basis vector, 13
Bravais flock, 14
Bravais lattice, 16
Bravais system, 16
Bravais type, 16

CARAT, 29
Cayley table, 10
Cell transformations, 430
Centralizer, 32
Centre of a rotoinversion, 6
Centre of inversion, 7
Centre of symmetry, 7
Centred lattice, 13
Centring translations
  additional, 43
  loss of, 43, 45, 48
Characteristic subgroup, 34
Closure, 9
Coefficients of a vector, 29
Column part, 7
Commutative group, 10
Complement, 36
Composition, law of, 9, 31
Composition series, 13
Congruence, 37
  trivial, 37
Conjugacy class, 11–12
Conjugate elements, 11
Conjugate subgroups, 12, 43, 45, 48, 51, 432
Conjugation action, 32
Continuum approach to phase transitions, 21
Continuum description, 20
Contracted graph, 54
Conventional basis, 7, 13
Conventional coordinate system, 13
Conventional setting, 43
Coordinate system, 7
  conventional, 13
  transformation of, 9, 46, 429
Coordinates, 7
Core of a subgroup, 32
Coset decomposition, 11
Coset length, 11
Coset representatives, 11
Cosets, left and right, 11, 32
Crystal, macroscopic, 6
Crystal class, 15
  geometric, 14
  holohedral, 16
  of space groups, 15
Crystal family, 14, 16
Crystal pattern, 6, 19
Crystal space, 29
Crystal structure, 35
  prediction of, 5
Crystal system, 14, 16
Crystallographic basis, 7
Crystallographic point group, 15
Crystallographic point orbit, 24
Crystallographic space-group type, 14
Crystallographic symmetry operation, 6, 9
Cyclic group, 10, 31

Daughter phase, 18
Deckoperation, 6
Decomposition of a group into cosets, 11
Decreased unit cell, 53
Deformed phase, 18
Derivative structures, 5, 25
Derived series, 37
Derived subgroup, 37
Detwinning, 19
Dimension of a vector space, 29
Direct product of two groups, 34
Direct space, 29
Distance, 31
Domain boundary, 19
Domain region, 19
Domain state, 19
  secondary, 20
Domain structure, 18
  translational, 20
Domain wall, 19
Domains, 18–19
  ferroelectric, 20

Elementary Abelian p-group, 36
Enantiomorphic space-group types, 14
Enantiomorphic space groups, 14, 18
Enlarged unit cell, 43, 49
Epimorphism, 10
Euclidean affine space, 31
Euclidean group, 18, 31
Euclidean metric, 31
Euclidean normalizer, 18, 24
  specialized, 18
  typical, 18
Euclidean vector space, 30

Factor group, 12, 33
Faithful action, 32
Faithful $\mathcal{G}$-set, 32
Ferroelastic phase transitions, 19
Ferroelectric domains, 20
Ferroelectric phase transitions, 25
Finite field, 33
Fixed point, 6

$\mathcal{G}$-set, 32
Galois, theorem of, 37
General position, 42
General subgroup, 18
General supergroup, 53
Generators, 10, 31, 42, 52
Geometric crystal class, 14
Glide plane, 7
Glide reflection, 7
Glide vector, 7
Graphs of group–subgroup relations, 54
  contracted, 54
  for *klassengleiche* subgroups, 55, 415
  for plane groups, 56
  for *translationengleiche* subgroups, 15, 54, 395
Group postulates, 9
Group table, 10
Group–subgroup relations between space groups, 16–18
  applications, 4–5, 24
  tree of, 5
Groups, 9, 31
  Abelian, 10, 31
  abstract, 10
  affine, 18, 30
  alternating, 34
  automorphism, 30
  commutative, 10
  cyclic, 10, 31
  Euclidean, 18, 31
  factor, 12, 33
  homomorphic, 10
  isomorphic, 10, 33
  linear, 30
  orthogonal, 18, 30
  plane, 13
  soluble, 37
  symmetric, 34
  translation, 7, 13, 18

Handedness, 6
Hermann, theorem of, 3, 22, 35
Hermann–Mauguin (HM) symbols, 42
Hermann's group, 21
Hettotype, 5, 25
Hexagonal axes, 43
Holohedral crystal class, 16
Holohedry, 16
Homomorphic groups, 10
Homomorphic image, 10, 12
Homomorphic mapping, 10
Homomorphism, 10, 12, 33
  injective, 33
  kernel of, 12, 33

Identity mapping (operation), 6
Image, homomorphic, 10, 12
Image point, 6
Index of a subgroup, 11, 23, 32, 429
Injective homomorphism, 33
Interchange of axes, 431–432
Interchange of Wyckoff labels, 430, 433
International symbols, 42
Invariant subgroup, 11
Inverse operation, 7, 10, 31
Inversion, 7, 13
Inversion centre, 7
Isometry, 6, 8, 31
Isomorphic $\mathcal{G}$-sets, 33
Isomorphic groups, 10, 33
Isomorphic subgroups, 4, 17, 23, 51, 56
  series of, 43, 51
Isomorphic supergroups, 53–54
Isomorphism, 10, 33–34
  multiple, 10
Isomorphism class, 10
Isomorphism theorems, 34

Isomorphism type, 10
Isosymbolic space group, 4

Kernel
  of a homomorphism, 12, 33
  of the action, 32
Klassengleiche (*k*-) subgroups, 3, 17, 35, 48, 415
Klassengleiche (*k*-) supergroups, 53

Lagrange, theorem of, 11, 32
Landau theory, 4
Lattice, 35
  Bravais, 16
  centred, 13
  primitive, 13
  vector, 7, 13
Lattice basis, 7
Lattice parameters, 8
Lattice system, 16
Lattice type, 16
Lattice vector, 7, 13
Law, associative, 10
Law of composition, 9, 31
Law of rational indices, 2
Law of symmetry, 2
Left coset, 11, 32
Length of a coset, 11
Linear group, 30
Linear mapping, 30
Linear part, 7, 30
Loss of centring translations, 43, 45, 48

Macroscopic crystal, 6
Macroscopic description, 20
Mapping
  affine, 6, 30
  homomorphic, 10
  identity, 6
  linear, 30
  orthogonal, 9
  reversible, 6
Matrix
  augmented, 8
  metric, 8
Matrix–column pair, 7
Matrix part, 7
Maximal-subgroup rule, 4
Maximal subgroups, 11, 23, 35, 36
  computation of, 27
Metric matrix, 8
Microscopic description, 20
Minimal supergroups, 11, 53
Mirror plane, 7
Monoclinic axis, 43
Monoclinic subgroups, settings of, 44, 52, 433
Motion, 6
  rigid, 6
Multiple isomorphism, 10
Multiplication table, 10
Multiplicity, 24–25

N-fold rotation, 6
N-fold rotoinversion, 6
Neutral element, 10
Nonconventional settings, 429–431
Non-ferroelastic phase transitions, 19
Non-isomorphic subgroups, 17, 23
Normal subgroups, 11–12, 33
Normalizers, 12, 33
  affine, 18, 24
  Euclidean, 18, 24

Orbit, 24, 32
Order of a group, 10, 31
Order of an element, 10
Order of listed subgroups, 43, 428
Order parameter, 4
Orientation state, 21
Orientational conjugation, 433
Origin choice, 13, 43–44
Origin shift, 9, 46–47, 51, 429–430
Orthogonal group, 18, 30
Orthogonal mapping, 9
Overlooked symmetry, 5

Paraelectric crystals, 25
Parent-clamping approximation (PCA), 18
Parent phase, 18
Periodicity, 6
Phase transitions, 4, 25
  continuum approach to, 21
  ferroelastic, 19
  ferroelectric, 25
  non-ferroelastic, 19
  reconstructive, 5
  second-order, 4
Plane group, 13
Point, fixed, 6
Point configuration, 24
Point group, crystallographic, 15
Point-group type, 15
Point space, 30
Position
  general, 42
  Wyckoff, 24, 430
Prediction of crystal structures, 5
Primitive basis, 7
Primitive $\mathcal{G}$-set, 37
Primitive lattice, 13
Product of group elements, 9, 31
Product of sets of group elements, 12
Proper subgroups, 11
Pseudosymmetry, 5
Punktlage, 24

Reconstructive phase transitions, 5
Reflection, 7
Reflection plane, 7
Region of a domain, 19
Representatives, coset, 11
Restrictions for indices, 429
Reversible mapping, 6
Rhombohedral axes, 43
Rhombohedral space groups, 45
Rhombohedral subgroups, settings of, 45
Right coset, 11, 32
Rigid motion, 6
Rotation, 6
  N-fold, 6
Rotation axis, 6
Rotoinversion, 6
  N-fold, 6
Rotoinversion axis, 7

Schoenflies symbols, 42
Screw axis, 6
Screw rotation, 6
Screw vector, 6
Second-order phase transitions, 4
Secondary domain state, 20
Seitz symbol, 7
Sequence of listed subgroups, 43, 428
Series of isomorphic subgroups, 43, 51
Set of generators, 10
Settings of monoclinic subgroups, 44, 52, 433
Settings of rhombohedral subgroups, 45
Single-domain state, 19
Single-domain structure, 19
Site symmetry, 24
Site-symmetry group, 15
Soluble group, 37
Space-group number, 42
Space-group types, 14
  affine, 14
  crystallographic, 14
  enantiomorphic, 14
  symmorphic, 15
Space groups, 13, 15, 35
  enantiomorphic, 14, 18
  isosymbolic, 4
  rhombohedral, 45
  symmorphic, 15, 36
Splitting of Wyckoff positions, 25, 430
Spontaneous deformation, 19
Spontaneous strain, 19
Stabilizer, 32
State
  domain, 19
  orientation, 21
  symmetry, 20
Subgroup graphs, 54, 395, 415
Subgroups, 11, 31
  characteristic, 34
  conjugate, 12, 43, 45, 48, 51, 432
  derived, 37
  general, 18
  invariant, 11
  isomorphic, 4, 17, 23, 51, 56
  klassengleiche, 3, 17, 35, 48, 415
  maximal, 11, 23, 35–36
  non-isomorphic, 17, 23
  normal, 11–12, 33
  of space groups, 16–18
  proper, 11
  translation, 31, 35
  translationengleiche, 3, 17, 23, 35, 45, 54, 395
  trivial, 12, 31
  zellengleiche, 3, 17
Supergroups, 11, 53
  general, 53
  isomorphic, 53–54
  klassengleiche, 53
  minimal, 11, 53
  translationengleiche, 53
Sylow, theorems of, 33
Sylow *p*-subgroup, 33
Symbols
  Hermann–Mauguin (HM), 42
  international, 42
  Schoenflies, 42
  Seitz, 7
Symmetric group, 34
Symmetry
  centre of, 7
  law of, 2
  overlooked, 5
Symmetry element, 9
Symmetry operation, 6
  crystallographic, 6, 9
Symmetry state, 20
Symmorphic space-group types, 15
Symmorphic space groups, 15, 36

Theorems
  Galois' theorem, 37
  Hermann's theorem, 3, 22, 35
  isomorphism theorems, 34
  Lagrange's theorem, 11, 32
  Sylow's theorems, 33
Transformation matrix, 47
Transformation of basis, 46–47, 51
Transformation of coordinate system, 9, 46, 429
Transitive $\mathcal{G}$-set, 32
Translation, 6, 13, 31
Translation group, 7, 13, 18

Translation part, 7
Translation subgroup, 31, 35
Translation twins, 20
Translation vector, 7
Translational conjugation, 432
Translational domain structure, 20
Translationengleiche (*t*-) subgroups, 3, 17, 23, 35, 45, 54, 395
Translationengleiche (*t*-) supergroups, 53
Transposed matrix, 9
Tree of group–subgroup relations, 5
Trivial congruence, 37
Trivial subgroup, 12, 31
Twinning, 5
Twins, 18–20

Underlying vector space, 30
Unit cell
  decreased, 53
  enlarged, 43, 49
Unit element, 10, 31

Vector, 8, 29
  basis, 13
  coefficients of, 29
  lattice, 7, 13
  translation, 7
Vector lattice, 7, 13
Vector space, 29
  dimension of, 29
  Euclidean, 30
  underlying, 30

Wyckoff labels, interchange of, 430, 433
Wyckoff positions, 24, 430
  splitting of, 25, 430
Wyckoff set, 24

Zellengleiche subgroups, 3, 17