International Tables for Crystallography (2006). Vol. A, Section 2.2.6, pp. 20-24.

2. GUIDE TO THE USE OF THE SPACE-GROUP TABLES

Table 2.2.5.1. Patterson symmetries for two and three dimensions

Laue class	Lat	Lattice type				Patterson symmetry (with space-group number)					
Two dimensions	5										
2	р					<i>p</i> 2 (2)					
2mm	р	с				<i>p</i> 2 <i>mm</i> (6)	<i>c</i> 2 <i>mm</i> (9)				
4	р					<i>p</i> 4 (10)					
4 <i>mm</i>	р					p4mm (11)					
6	р					<i>p</i> 6 (16)					
6 <i>mm</i>	р					<i>p</i> 6 <i>mm</i> (17)					
Three dimensions											
ī	Р					$P\bar{1}$ (2)					
2/m	Р	С				P2/m (10)	C2/m (12)				
mmm	Р	С	Ι	F		Pmmm (47)	Cmmm (65)	Immm (71)	Fmmm (69)		
4/m	Р		Ι			P4/m (83)		I4/m (87)			
4/mmm	Р		Ι			P4/mmm (123)		I4/mmm (139)			
3	Р				R	P3 (147)				<i>R</i> 3 (148)	
$\int \bar{3}m1$	Р				R	<i>P</i> 3 <i>m</i> 1 (164)				$R\bar{3}m$ (166)	
$\left\{ \bar{3}1m\right.$	Р					$P\bar{3}1m$ (162)					
6/m	Р					P6/m (175)					
6/mmm	Р					P6/mmm (191)					
m3	Р		Ι	F		$Pm\bar{3}$ (200)		$Im\bar{3}$ (204)	$Fm\bar{3}$ (202)		
m3m	Р		Ι	F		$Pm\bar{3}m$ (221)		$Im\bar{3}m$ (229)	$Fm\bar{3}m$ (225)		

2.2.6. Space-group diagrams

The space-group diagrams serve two purposes: (i) to show the relative locations and orientations of the symmetry elements and (ii) to illustrate the arrangement of a set of symmetrically equivalent points of the general position.

All diagrams are orthogonal projections, *i.e.* the projection direction is perpendicular to the plane of the figure. Apart from the descriptions of the rhombohedral space groups with 'rhombohedral axes' (*cf.* Section 2.2.6.6), the projection direction is always a cell axis. If other axes are not parallel to the plane of the figure, they are indicated by the subscript *p*, as a_p , b_p or c_p . This applies to one or two axes for triclinic and monoclinic space groups (*cf.* Figs. 2.2.6.1 to 2.2.6.3), as well as to the three rhombohedral axes in Fig. 2.2.6.9.

The graphical symbols for symmetry elements, as used in the drawings, are displayed in Chapter 1.4.

In the diagrams, 'heights' *h* above the projection plane are indicated for symmetry planes and symmetry axes *parallel* to the projection plane, as well as for centres of symmetry. The heights are given as fractions of the shortest lattice translation normal to the projection plane and, if different from 0, are printed next to the graphical symbols. Each symmetry element at height *h* is accompanied by another symmetry element of the same type at height $h + \frac{1}{2}$ (this does not apply to the horizontal fourfold axes in the cubic diagrams). In the space-group diagrams, only the symmetry element at height *h* is indicated (*cf.* Chapter 1.4).

Schematic representations of the diagrams, displaying the origin, the labels of the axes, and the projection direction [uvw], are given in Figs. 2.2.6.1 to 2.2.6.10 (except Fig. 2.2.6.6). The general-position diagrams are indicated by the letter G.

2.2.6.1. Plane groups

Each description of a plane group contains two diagrams, one for the symmetry elements (left) and one for the general position (right). The two axes are labelled a and b, with a pointing downwards and b running from left to right.

2.2.6.2. Triclinic space groups

For each of the two triclinic space groups, three elevations (along a, b and c) are given, in addition to the general-position diagram G (projected along c) at the lower right of the set, as illustrated in Fig. 2.2.6.1.

The diagrams represent a reduced cell of type II for which the three interaxial angles are non-acute, *i.e.* $\alpha, \beta, \gamma \ge 90^{\circ}$. For a cell of type I, all angles are acute, *i.e.* $\alpha, \beta, \gamma < 90^{\circ}$. For a discussion of the two types of reduced cells, reference is made to Section 9.2.2.

2.2.6.3. *Monoclinic space groups (cf. Sections 2.2.2 and 2.2.16)*

The 'complete treatment' of each of the two settings contains four diagrams (Figs. 2.2.6.2 and 2.2.6.3). Three of them are projections of the symmetry elements, taken along the unique axis (upper left) and along the other two axes (lower left and upper right). For the general position, only the projection along the unique axis is given (lower right).

The 'synoptic descriptions' of the three cell choices (for each setting) are headed by a pair of diagrams, as illustrated in Fig. 2.2.6.4. The drawings on the left display the symmetry elements and the ones on the right the general position (labelled G). Each diagram is a projection of four neighbouring unit cells along the unique axis. It contains the outlines of the three cell choices drawn as heavy lines. For the labelling of the axes, see Fig. 2.2.6.4. The headline of the description of each cell choice contains a small-scale drawing, indicating the basis vectors and the cell that apply to that description.

2.2.6.4. Orthorhombic space groups and orthorhombic settings

The space-group tables contain a set of four diagrams for each orthorhombic space group. The set consists of three projections of the symmetry elements [along the c axis (upper left), the a axis (lower left) and the b axis (upper right)] in addition to the general-position diagram, which is given only in the projection along c

2.2. CONTENTS AND ARRANGEMENT OF THE TABLES

Table 2.2.6.1. Numbers of distinct projections and different
Hermann–Mauguin symbols for the orthorhombic space
groups (space-group number placed between parentheses),
listed according to point group as indicated in the headline

Number of			
distinct			
projections	222	mm2	$2/m^2/m^2/m$
6		$Pmc2_1$ (26)	$P 2_1/m 2/m 2/a (51)$
(22 space groups)		Pma2 (28)	$P 2/n 2_1/n 2/a$ (52)
		$Pca2_1$ (29)	$P \ 2/m \ 2/n \ 2_1/a \ (53)$
		Pnc2 (30)	$P 2_1/c 2/c 2/a$ (54)
		$\textit{Pmn2}_1 \ (31)$	$P \ 2/b \ 2_1/c \ 2_1/m \ (57)$
		$Pna2_1$ (33)	$P 2_1/b 2/c 2_1/n$ (60)
		$Cmc2_1$ (36)	$P 2_1/n 2_1/m 2_1/a$ (62)
		Amm2 (38)	$C \ 2/m \ 2/c \ 2_1/m \ (63)$
		Abm2 (39)	$C \ 2/m \ 2/c \ 2_1/a \ (64)$
		Ama2 (40)	$I 2_1/m 2_1/m 2_1/a$ (74)
		Aba2 (41)	
		Ima2 (46)	
3	P222 ₁ (17)	Pmm2 (25)	$P \ 2/c \ 2/c \ 2/m \ (49)$
(25 space groups)	$P2_12_12$ (18)	Pcc2 (27)	P 2/b 2/a 2/n (50)
	$C222_1$ (20)	Pba2 (32)	$P 2_1/b 2_1/a 2/m$ (55)
	C222 (21)	Pnn2 (34)	$P 2_1/c 2_1/c 2/n$ (56)
		Cmm2 (35)	$P 2_1/n 2_1/n 2/m$ (58)
		Ccc2 (37)	$P 2_1/m 2_1/m 2/n$ (59)
		Fmm2 (42)	C 2/m 2/m 2/m (65)
		Fdd2 (43)	$C \ 2/c \ 2/c \ 2/m \ (66)$
		Imm2 (44)	C 2/m 2/m 2/a (67)
		Iba2 (45)	$C \ 2/c \ 2/c \ 2/a \ (68)$
			$I \ 2/b \ 2/a \ 2/m \ (72)$
2			$P 2_1/b 2_1/c 2_1/a$ (61)
(2 space groups)			$I 2_1/b 2_1/c 2_1/a$ (73)
1	P222 (16)		$P \ 2/m \ 2/m \ 2/m \ (47)$
(10 space groups)	$P2_12_12_1$ (19)		$P \ 2/n \ 2/n \ 2/n \ (48)$
	F222 (22)		$F \ 2/m \ 2/m \ 2/m \ (69)$
	<i>I</i> 222 (23)		$F \ 2/d \ 2/d \ 2/d \ (70)$
	$I2_12_12_1$ (24)		$I \ 2/m \ 2/m \ 2/m \ (71)$
Total: (59)	(9)	(22)	(28)

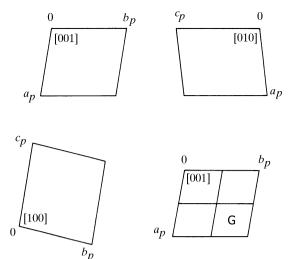


Fig. 2.2.6.1. Triclinic space groups (G = general-position diagram).

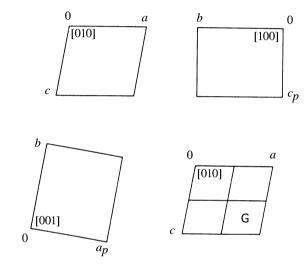


Fig. 2.2.6.2. Monoclinic space groups, setting with unique axis b (G = general-position diagram).

(lower right). The projected axes, the origins and the projection directions of these diagrams are illustrated in Fig. 2.2.6.5. They refer to the so-called 'standard setting' of the space group, *i.e.* the setting described in the space-group tables and indicated by the 'standard Hermann–Mauguin symbol' in the headline.

For each orthorhombic space group, six settings exist, *i.e.* six different ways of assigning the labels a, b, c to the three orthorhombic symmetry directions; thus the shape and orientation of the cell are the same for each setting. These settings correspond to the six permutations of the labels of the axes (including the identity permutation); cf. Section 2.2.16:

abc ba \overline{c} cab \overline{c} ba bca a \overline{c} b.

The symbol for each setting, here called 'setting symbol', is a shorthand notation for the transformation of the basis vectors of the

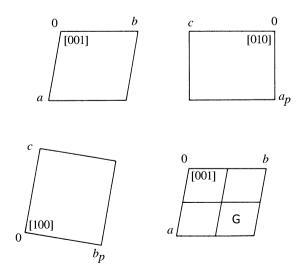


Fig. 2.2.6.3. Monoclinic space groups, setting with unique axis c.

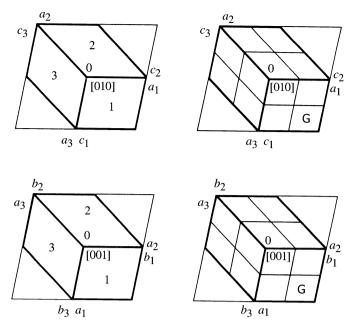


Fig. 2.2.6.4. Monoclinic space groups, cell choices 1, 2, 3. Upper pair of diagrams: setting with unique axis *b*. Lower pair of diagrams: setting with unique axis *c*. The numbers 1, 2, 3 within the cells and the subscripts of the labels of the axes indicate the cell choice (*cf.* Section 2.2.16). The unique axis points upwards from the page.

standard setting, **a**, **b**, **c**, into those of the setting considered. For instance, the setting symbol **cab** stands for the cyclic permutation

$$\mathbf{a}' = \mathbf{c}, \quad \mathbf{b}' = \mathbf{a}, \quad \mathbf{c}' = \mathbf{b}$$

or

$$(\mathbf{a}'\mathbf{b}'\mathbf{c}') = (\mathbf{abc}) \begin{pmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0 \end{pmatrix} = (\mathbf{cab}),$$

where $\mathbf{a}', \mathbf{b}', \mathbf{c}'$ is the new set of basis vectors. An interchange of two axes reverses the handedness of the coordinate system; in order to keep the system right-handed, each interchange is accompanied by the reversal of the sense of one axis, *i.e.* by an element $\overline{1}$ in the transformation matrix. Thus, **ba** $\overline{\mathbf{c}}$ denotes the transformation

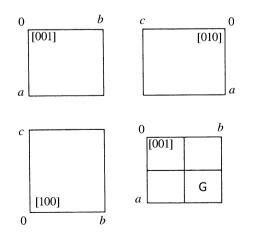


Fig. 2.2.6.5. Orthorhombic space groups. Diagrams for the 'standard setting' as described in the space-group tables (G = general-position diagram).

$$(\mathbf{a}'\mathbf{b}'\mathbf{c}') = (\mathbf{a}\mathbf{b}\mathbf{c}) \begin{pmatrix} 0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & \overline{1} \end{pmatrix} = (\mathbf{b}\mathbf{a}\overline{\mathbf{c}})$$

The six orthorhombic settings correspond to six Hermann–Mauguin symbols which, however, need not all be different; cf. Table 2.2.6.1.*

In the earlier (1935 and 1952) editions of *International Tables*, only one setting was illustrated, in a projection along c, so that it was usual to consider it as the 'standard setting' and to accept its cell edges as crystal axes and its space-group symbol as 'standard Hermann–Mauguin symbol'. In the present edition, however, *all six* orthorhombic settings are illustrated, as explained below.

The three projections of the symmetry elements can be interpreted in two ways. First, in the sense indicated above, that is, as different projections of a single (standard) setting of the space group, with the projected basis vectors **a**, **b**, **c** labelled as in Fig. 2.2.6.5. Second, each one of the three diagrams can be considered as the projection along \mathbf{c}' of either one of *two different* settings: one setting in which \mathbf{b}' is horizontal and one in which \mathbf{b}' is vertical $(\mathbf{a}', \mathbf{b}', \mathbf{c}')$ refer to the setting under consideration). This second interpretation is used to illustrate in the same figure the space-group symbols corresponding to these two settings. In order to view these projections in conventional orientation (\mathbf{b}' horizontal. \mathbf{a}' vertical. origin in the upper left corner, projection down the positive \mathbf{c}' axis), the setting with \mathbf{b}' horizontal can be inspected directly with the figure upright; hence, the corresponding space-group symbol is printed above the projection. The other setting with \mathbf{b}' vertical and \mathbf{a}' horizontal, however, requires turning the figure over 90°, or looking at it from the side; thus, the space-group symbol is printed at the left, and it runs upwards.

The 'setting symbols' for the six settings are attached to the three diagrams of Fig. 2.2.6.6, which correspond to those of Fig. 2.2.6.5. In the orientation of the diagram where the setting symbol is read in the usual way, \mathbf{a}' is vertical pointing downwards, \mathbf{b}' is horizontal pointing to the right, and \mathbf{c}' is pointing upwards from the page. Each setting symbol is printed in the position that in the space-group tables is actually occupied by the corresponding full Hermann-Mauguin symbol. The changes in the space-group symbol that are

^{*} A space-group symbol is invariant under sign changes of the axes; *i.e.* the same symbol applies to the right-handed coordinate systems \mathbf{abc} , \mathbf{abc} , $\mathbf{\overline{abc}}$, $\mathbf{\overline{abc}}$ and the left-handed systems $\mathbf{\overline{abc}}$, \mathbf{abc} , \mathbf{abc} , $\mathbf{\overline{abc}}$.

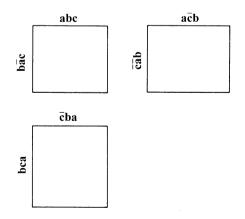


Fig. 2.2.6.6. Orthorhombic space groups. The three projections of the symmetry elements with the six setting symbols (see text). For setting symbols printed vertically, the page has to be turned clockwise by 90° or viewed from the side. Note that in the actual space-group tables instead of the setting symbols the corresponding full Hermann–Mauguin space-group symbols are printed.

associated with a particular setting symbol can easily be deduced by comparing Fig. 2.2.6.6 with the diagrams for the space group under consideration.

Not all of the 59 orthorhombic space groups have all six projections distinct, *i.e.* have different Hermann–Mauguin symbols for the six settings. This aspect is treated in Table 2.2.6.1. Only 22 space groups have six, 25 have three, 2 have two different symbols, while 10 have all symbols the same. This information can be of help in the early stages of a crystal-structure analysis.

The six setting symbols listed in the second paragraph of this section form the column headings of the orthorhombic entries in Table 4.3.2.1, which contains the extended Hermann–Mauguin symbols for the six settings of each orthorhombic space group. Note that some of these setting symbols exhibit different sign changes compared with those in Fig. 2.2.6.6.

2.2.6.5. Tetragonal, trigonal P and hexagonal P space groups

The pairs of diagrams for these space groups are similar to those in IT (1935) and IT (1952). Each pair consists of a general-position diagram (right) and a diagram of the symmetry elements (left), both projected along c, as illustrated in Figs. 2.2.6.7 and 2.2.6.8.

2.2.6.6. Rhombohedral (trigonal R) space groups

The seven rhombohedral R space groups are treated in two versions, the first based on 'hexagonal axes' (obverse setting), the second on 'rhombohedral axes' (*cf.* Sections 2.1.3 and 2.2.2). The pairs of diagrams are similar to those in *IT* (1952); the left or top one displays the symmetry elements, the right or bottom one the general position. This is illustrated in Fig. 2.2.6.9, which gives the axes a and b of the triple hexagonal cell and the projections of the axes of the primitive rhombohedral cell, labelled a_p , b_p and c_p . For convenience, all 'heights' in the space-group diagrams are fractions of the hexagonal c axis. For 'hexagonal axes', the projection direction is [001], for 'rhombohedral axes' it is [111]. In the general-position diagrams, the circles drawn in heavier lines represent atoms that lie within the primitive rhombohedral cell (provided the symbol '-' is read as 1 - z rather than as -z).

The pairs of drawings for the hexagonal and the rhombohedral descriptions of a space group are the same. In the rhombohedral descriptions of space groups Nos. 166 and 167, $R\bar{3}m$ and $R\bar{3}c$, the diagrams are omitted for reasons of space, and the reader is referred to the drawings in the hexagonal descriptions.

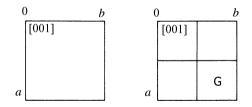


Fig. 2.2.6.7. Tetragonal space groups (G = general-position diagram).

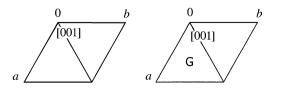


Fig. 2.2.6.8. Trigonal P and hexagonal P space groups (G = general-position diagram).

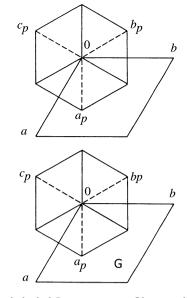


Fig. 2.2.6.9. Rhombohedral *R* space groups. Obverse triple hexagonal cell with 'hexagonal axes' *a*, *b* and primitive rhombohedral cell with projections of 'rhombohedral axes' a_p, b_p, c_p . Note: In the actual space-group diagrams only the upper edges (full lines), not the lower edges (dashed lines) of the primitive rhombohedral cell are shown (G = general-position diagram).

2.2.6.7. Cubic space groups

For each cubic space group, one projection of the symmetry elements along [001] is given, Fig. 2.2.6.10; for details of the diagrams, see Chapter 1.4 and Buerger (1956). For face-centred lattices F, only a quarter of the unit cell is shown; this is sufficient since the projected arrangement of the symmetry elements is translation-equivalent in the four quarters of an F cell. The three stereoscopic general-position diagrams in the lower part of the page are explained below.

The cubic diagrams given in IT (1935) were quite different from the ones used here. No drawings for cubic space groups were provided in IT (1952).

2.2.6.8. Diagrams of the general position

(i) Non-cubic space groups

In these diagrams, the 'heights' of the points are z coordinates, except for monoclinic space groups with unique axis b where they are y coordinates. For rhombohedral space groups, the heights are always fractions of the hexagonal c axis. The symbols + and - stand for +z and -z (or +y and -y) in which z or y can assume any value. For points with symbols + or - preceded by a fraction, *e.g.*

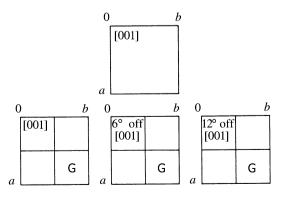


Fig. 2.2.6.10. Cubic space groups (G = general-position stereodiagrams).

 $\frac{1}{2}$ + or $\frac{1}{3}$ -, the relative *z* or *y* coordinate is $\frac{1}{2}$ *etc.* higher than that of the point with symbol + or -.

Points represented by \bigcirc and \odot are related by inversion, rotoinversion or mirror symmetry and are thus enantiomorphs of each other. If \bigcirc were to be occupied by the centre of a right-handed molecule, the molecule at \bigcirc would be left-handed.

Where a mirror plane exists parallel to the plane of projection, the two positions superimposed in projection are indicated by the use of a ring divided through the centre. The information given on each side refers to one of the two positions related by the mirror plane, as in - +.

(ii) Stereodiagrams for cubic space groups (Fig. 2.2.6.10)

For each cubic space group, three diagrams are given with the points of the general position as vertices of transparent polyhedra. (The spheres at the vertices are depicted as opaque, however.) For the 'starting point', the same coordinate values, x = 0.048, y = 0.12, z = 0.08, as in the cubic diagrams of *IT* (1935) have been used. The diagram on the left corresponds to that published in *IT* (1935); in this figure, the height *h* of the centre of each polyhedron is given, if different from zero. For space groups Nos. 198, 199 and 220, *h* refers to the special point to which the polyhedron (triangle) is connected by dotted lines. In all diagrams, polyhedra with height 1 are omitted. A grid of four squares is drawn to represent the four quarters of the basal plane of the cell.

Of the three diagrams, the image on the left and the central one form a stereopair, as well as the central image and that on the right (Langlet, 1972). The stereoscopic effect is obtained by a 6° tilt between each view. The separation of neighbouring images has the standard value of 55 mm. The presence of the two stereopairs has the advantage that difficulties in seeing the polyhedra due to overlap in one pair do not occur in the other. For space groups Nos. 219, 226 and 228, where the number of points was too large for one set, two sets of three drawings are provided, one for the upper and one for the lower half of the cell.

Notes:

- (i) For space group $P4_132$ (213), the coordinates $\bar{x}, \bar{y}, \bar{z}$ have been chosen for the 'starting point' to bring out the enantiomorphism with $P4_332$ (212).
- (ii) For the description of a space group with 'Origin choice 2', the coordinates x, y, z of all points have been shifted with the origin to retain the same polyhedra for both origin choices.

Readers who wish to compare other approaches to space-group diagrams and their history are referred to *IT* (1935), *IT* (1952) and the following publications: Astbury & Yardley (1924); Belov *et al.* (1980); Buerger (1956); Fedorov (1895; English translation, 1971); Friedel (1926); Hilton (1903); Niggli (1919); Schiebold (1929).

2.2.7. Origin

The determination and description of crystal structures and particularly the application of direct methods are greatly facilitated by the choice of a suitable origin and its proper identification. This is even more important if related structures are to be compared or if 'chains' of group–subgroup relations are to be constructed. In this volume, as well as in *IT* (1952), the origin of the unit cell has been chosen according to the following conventions (*cf.* Chapter 2.1 and Section 2.2.2):

(i) All centrosymmetric space groups are described with an inversion centre as origin. A further description is given if a centrosymmetric space group contains points of high site symmetry that do not coincide with a centre of symmetry.

Example: $I4_1/amd$ (141).

(ii) For noncentrosymmetric space groups, the origin is at a point of highest site symmetry, as in $P\bar{6}m2$ (187). If no site symmetry is higher than 1, except for the cases listed below under (iii), the origin is placed on a screw axis, or a glide plane, or at the intersection of several such symmetry elements.

Examples: Pca2₁ (29); P6₁ (169).

(iii) In space group $P2_12_12_1$ (19), the origin is chosen in such a way that it is surrounded symmetrically by three pairs of 2_1 axes. This principle is maintained in the following noncentrosymmetric cubic space groups of classes 23 and 432, which contain $P2_12_12_1$ as subgroup: $P2_13$ (198), $I2_13$ (199), $F4_132$ (210). It has been extended to other noncentrosymmetric orthorhombic and cubic space groups with $P2_12_12_1$ as subgroup, even though in these cases points of higher site symmetry *are* available: $I2_12_12_1$ (24), $P4_332$ (212), $P4_132$ (213), $I4_132$ (214).

There are several ways of determining the location and site symmetry of the origin. First, the origin can be inspected directly in the space-group diagrams (*cf.* Section 2.2.6). This method permits visualization of all symmetry elements that intersect the chosen origin.

Another procedure for finding the site symmetry at the origin is to look for a special position that contains the coordinate triplet 0, 0, 0 or that includes it for special values of the parameters, *e.g.* position 1*a*: 0, 0, *z* in space group *P*4 (75), or position $3a : x, 0, \frac{1}{3}$; 0, *x*, $\frac{2}{3}$; $\bar{x}, \bar{x}, 0$ in space group *P*3₁21 (152). If such a special position occurs, the symmetry at the origin is given by the oriented site-symmetry symbol (see Section 2.2.12) of that special position; if it does not occur, the site symmetry at the origin is 1. For most practical purposes, these two methods are sufficient for the identification of the site symmetry at the origin.

2.2.7.1. Origin statement

In the line *Origin* immediately below the diagrams, the site symmetry of the origin is stated, if different from the identity. A further symbol indicates all symmetry elements (including glide planes and screw axes) that pass through the origin, if any. For space groups with two *origin choices*, for each of the two origins the location relative to the other origin is also given. An example is space group *Ccca* (68).

In order to keep the notation as simple as possible, no rigid rules have been applied in formulating the origin statements. Their meaning is demonstrated by the examples in Table 2.2.7.1, which should be studied together with the appropriate space-group diagrams.

These examples illustrate the following points:

(i) The site symmetry at the origin corresponds to the point group of the space group (examples E1-E3) or to a subgroup of this point group (E4-E11).

The presence of a symmetry centre at the origin is always stated explicitly, either by giving the symbol $\overline{1}$ (*E*1 and *E*4) or by the words 'at centre', followed by the full site symmetry between parentheses (*E*2 and *E*5). This completes the origin line, if no further glide planes or screw axes are present at the origin.

(ii) If glide planes or screw axes are present, as in examples E4-E11, they are given in the order of the symmetry directions listed in Table 2.2.4.1. Such a set of symmetry elements is described here in the form of a 'point-group-like' symbol (although it does not describe a group). With the help of the orthorhombic symmetry directions, the symbols in E4-E6 can be interpreted easily. The shortened notation of E6 and E7 is used for space groups of crystal classes mm2, 4mm, 42m, 3m, 6mm and 62m if the site symmetry at the origin can be easily recognized from the shortened symbol.