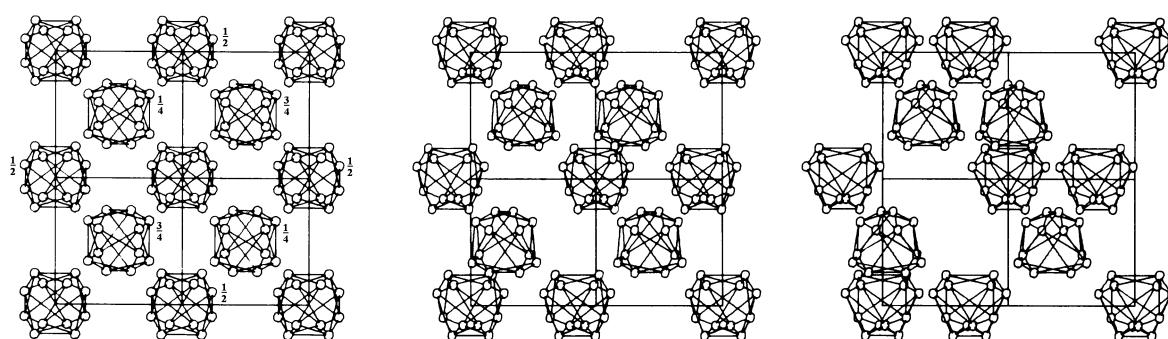
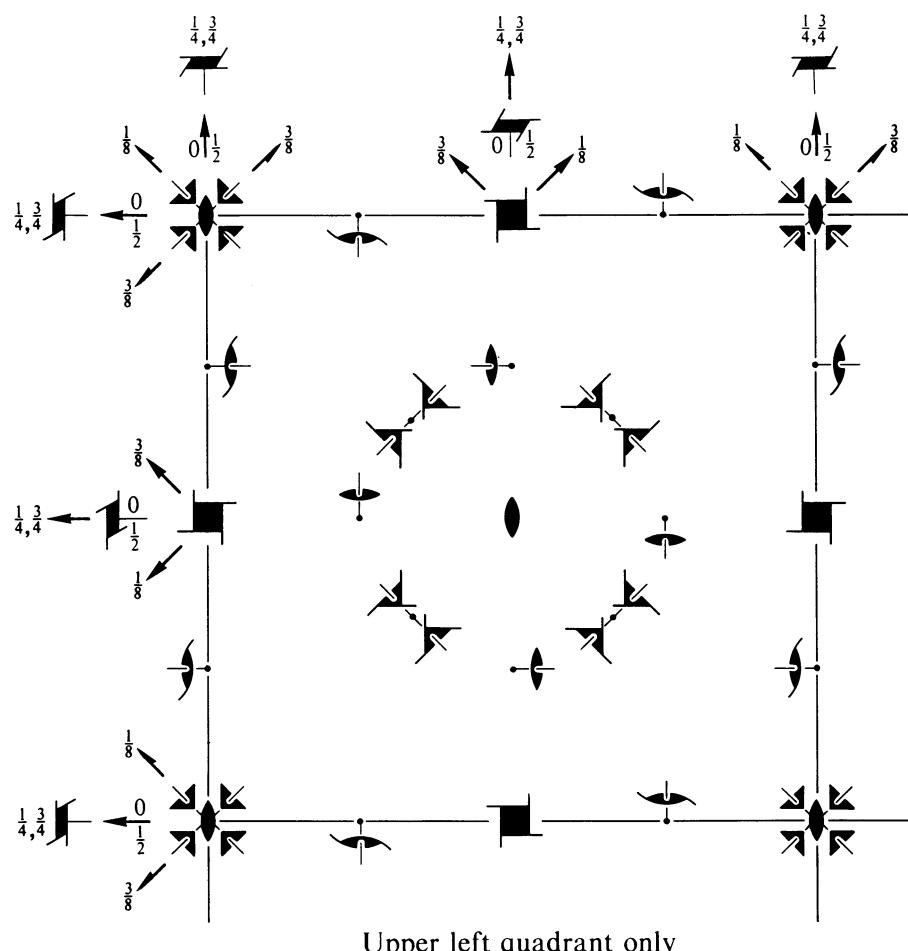


$F4_132$ O^4

432

Cubic

No. 210

 $F4_132$ Patterson symmetry $Fm\bar{3}m$ 

Origin at 23

Asymmetric unit $0 \leq x \leq \frac{1}{2}; -\frac{1}{8} \leq y \leq \frac{1}{8}; -\frac{1}{8} \leq z \leq \frac{1}{8}; y \leq \min(x, \frac{1}{2} - x); -y \leq z \leq \min(x, \frac{1}{2} - x)$

Vertices	$0, 0, 0$	$\frac{1}{8}, \frac{1}{8}, \frac{1}{8}$	$\frac{1}{8}, \frac{1}{8}, -\frac{1}{8}$	$\frac{1}{8}, -\frac{1}{8}, \frac{1}{8}$
	$\frac{1}{2}, 0, 0$	$\frac{3}{8}, \frac{1}{8}, \frac{1}{8}$	$\frac{3}{8}, \frac{1}{8}, -\frac{1}{8}$	$\frac{3}{8}, -\frac{1}{8}, \frac{1}{8}$

Symmetry operations

For $(0,0,0)$ + set

- | | | | |
|---------------------------------------|---|---|--|
| (1) 1 | (2) $2(0,0,\frac{1}{2})$ | (3) $2(0,\frac{1}{2},0)$ | (4) $2(\frac{1}{2},0,0)$ |
| (5) $3^+ x, x, x$ | (6) $3^+(\frac{1}{3}, -\frac{1}{3}, \frac{1}{3})$ | (7) $3^+(-\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ | (8) $3^+(\frac{1}{3}, \frac{1}{3}, -\frac{1}{3})$ |
| (9) $3^- x, x, x$ | (10) $3^- x, \bar{x} + \frac{1}{2}, \bar{x}$ | (11) $3^- \bar{x} + \frac{1}{2}, \bar{x}, x$ | (12) $3^- \bar{x} - \frac{1}{2}, x + \frac{1}{2}, \bar{x}$ |
| (13) $2(\frac{1}{2}, \frac{1}{2}, 0)$ | (14) $2 x, \bar{x} + \frac{1}{4}, \frac{1}{8}$ | (15) $4^-(0,0,\frac{3}{4})$ | (16) $4^+(0,0,\frac{1}{4})$ |
| (17) $4^-(\frac{1}{4}, 0, 0)$ | (18) $2(0, \frac{1}{2}, \frac{1}{2})$ | (19) $2 \frac{1}{8} y + \frac{1}{4}, \bar{y}$ | (20) $4^+(\frac{1}{4}, 0, 0)$ |
| (21) $4^+(\frac{1}{4}, 0, 0)$ | (22) $2(\frac{1}{2}, 0, \frac{1}{2})$ | (23) $4^-(0, \frac{3}{4}, 0)$ | (24) $2 \bar{x} + \frac{1}{4}, \frac{1}{8}, x$ |
| | | | |

Symmetry operations (*continued*)For $(0, \frac{1}{2}, \frac{1}{2})$ + set

- (1) $t(0, \frac{1}{2}, \frac{1}{2})$
 (5) $3^+(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) x - \frac{1}{3}, x - \frac{1}{6}, x$
 (9) $3^-(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) x - \frac{1}{6}, x + \frac{1}{6}, x$
 (13) $2(\frac{3}{4}, \frac{3}{4}, 0) x, x, \frac{1}{8}$
 (17) $4^-(\frac{3}{4}, 0, 0) x, \frac{1}{4}, -\frac{1}{4}$
 (21) $4^+(\frac{3}{4}, 0) \frac{1}{2}, y, -\frac{1}{4}$
- (2) $2 0, 0, z$
 (6) $3^+ \bar{x} + \frac{1}{2}, x, \bar{x}$
 (10) $3^- x + \frac{1}{2}, \bar{x}, \bar{x}$
 (14) $2(-\frac{1}{4}, \frac{1}{4}, 0) x, \bar{x} + \frac{1}{2}, \frac{3}{8}$
 (18) $2(0, \frac{1}{2}, \frac{1}{2}) \frac{3}{8}, y - \frac{1}{4}, y$
 (22) $2(\frac{1}{4}, 0, \frac{1}{4}) x, \frac{1}{8}, x$

For $(\frac{1}{2}, 0, \frac{1}{2})$ + set

- (1) $t(\frac{1}{2}, 0, \frac{1}{2})$
 (5) $3^+(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) x + \frac{1}{6}, x - \frac{1}{6}, x$
 (9) $3^-(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) x - \frac{1}{6}, x - \frac{1}{3}, x$
 (13) $2(\frac{1}{4}, \frac{1}{4}, 0) x, x, \frac{1}{8}$
 (17) $4^-(\frac{1}{4}, 0, 0) x, \frac{1}{4}, 0$
 (21) $4^+(\frac{1}{4}, 0) \frac{1}{4}, y, 0$
- (2) $2 \frac{1}{4}, \frac{1}{4}, z$
 (6) $3^+ \bar{x}, x, \bar{x}$
 (10) $3^- (-\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) x + \frac{1}{6}, \bar{x} + \frac{1}{6}, \bar{x}$
 (14) $2(\frac{1}{4}, -\frac{1}{4}, 0) x, \bar{x} + \frac{1}{2}, \frac{3}{8}$
 (18) $2(0, \frac{3}{4}, \frac{3}{4}) \frac{1}{8}, y, y$
 (22) $2(\frac{1}{2}, 0, \frac{1}{2}) x + \frac{1}{4}, \frac{3}{8}, x$

For $(\frac{1}{2}, \frac{1}{2}, 0)$ + set

- (1) $t(\frac{1}{2}, \frac{1}{2}, 0)$
 (5) $3^+(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) x + \frac{1}{6}, x + \frac{1}{3}, x$
 (9) $3^-(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) x + \frac{1}{3}, x + \frac{1}{6}, x$
 (13) $2(\frac{1}{2}, \frac{1}{2}, 0) x, x + \frac{1}{4}, \frac{3}{8}$
 (17) $4^-(\frac{1}{4}, 0, 0) x, \frac{3}{4}, 0$
 (21) $4^+(\frac{1}{4}, 0) \frac{1}{2}, y, \frac{1}{4}$
- (2) $2(0, 0, \frac{1}{2}) \frac{1}{4}, 0, z$
 (6) $3^+ \bar{x}, x + \frac{1}{2}, \bar{x}$
 (10) $3^- x, \bar{x}, \bar{x}$
 (14) $2 x, \bar{x} + \frac{3}{4}, \frac{1}{8}$
 (18) $2(0, \frac{1}{4}, \frac{1}{4}) \frac{1}{8}, y, y$
 (22) $2(\frac{1}{4}, 0, \frac{1}{4}) x, \frac{1}{8}, x$

Generators selected (1); $t(1, 0, 0)$; $t(0, 1, 0)$; $t(0, 0, 1)$; $t(0, \frac{1}{2}, \frac{1}{2})$; $t(\frac{1}{2}, 0, \frac{1}{2})$; (2); (3); (5); (13)

Positions

Multiplicity,
Wyckoff letter,
Site symmetry

Coordinates

 $(0, 0, 0) + (0, \frac{1}{2}, \frac{1}{2}) + (\frac{1}{2}, 0, \frac{1}{2}) + (\frac{1}{2}, \frac{1}{2}, 0) +$

Reflection conditions

 h, k, l permutable

General:

- 96 h 1 (1) x, y, z
 (5) z, x, y
 (9) y, z, x
 (13) $y + \frac{3}{4}, x + \frac{1}{4}, \bar{z} + \frac{3}{4}$
 (17) $x + \frac{3}{4}, z + \frac{1}{4}, \bar{y} + \frac{3}{4}$
 (21) $z + \frac{3}{4}, y + \frac{1}{4}, \bar{x} + \frac{3}{4}$
- (2) $\bar{x}, \bar{y} + \frac{1}{2}, z + \frac{1}{2}$
 (6) $z + \frac{1}{2}, \bar{x}, \bar{y} + \frac{1}{2}$
 (10) $\bar{y} + \frac{1}{2}, z + \frac{1}{2}, \bar{x}$
 (14) $\bar{y} + \frac{1}{4}, \bar{x} + \frac{1}{4}, \bar{z} + \frac{1}{4}$
 (18) $\bar{x} + \frac{3}{4}, z + \frac{3}{4}, y + \frac{1}{4}$
 (22) $z + \frac{1}{4}, \bar{y} + \frac{3}{4}, x + \frac{3}{4}$
- (3) $\bar{x} + \frac{1}{2}, y + \frac{1}{2}, \bar{z}$
 (7) $\bar{z}, \bar{x} + \frac{1}{2}, y + \frac{1}{2}$
 (11) $y + \frac{1}{2}, \bar{z}, \bar{x} + \frac{1}{2}$
 (15) $y + \frac{1}{4}, \bar{x} + \frac{3}{4}, z + \frac{3}{4}$
 (19) $\bar{x} + \frac{1}{4}, \bar{z} + \frac{1}{4}, \bar{y} + \frac{1}{4}$
 (23) $\bar{z} + \frac{3}{4}, y + \frac{3}{4}, x + \frac{1}{4}$
- (4) $x + \frac{1}{2}, \bar{y}, \bar{z} + \frac{1}{2}$
 (8) $\bar{z} + \frac{1}{2}, x + \frac{1}{2}, \bar{y}$
 (12) $\bar{y}, \bar{z} + \frac{1}{2}, x + \frac{1}{2}$
 (16) $\bar{y} + \frac{3}{4}, x + \frac{3}{4}, z + \frac{1}{4}$
 (20) $x + \frac{1}{4}, \bar{z} + \frac{3}{4}, y + \frac{3}{4}$
 (24) $\bar{z} + \frac{1}{4}, \bar{y} + \frac{1}{4}, \bar{x} + \frac{1}{4}$

Special: as above, plus

- 48 g .. 2 $\frac{1}{8}, y, \bar{y} + \frac{1}{4}$
 $\bar{y} + \frac{1}{4}, \frac{1}{8}, y$
 $y, \bar{y} + \frac{1}{4}, \frac{1}{8}$
- $\frac{7}{8}, \bar{y} + \frac{1}{2}, \bar{y} + \frac{3}{4}$
 $\bar{y} + \frac{3}{4}, \frac{7}{8}, \bar{y} + \frac{1}{2}$
 $\bar{y} + \frac{1}{2}, \bar{y} + \frac{3}{4}, \frac{7}{8}$
- $\frac{3}{8}, y + \frac{1}{2}, y + \frac{3}{4}$
 $y + \frac{3}{4}, \frac{3}{8}, y + \frac{1}{2}$
 $y + \frac{1}{2}, y + \frac{3}{4}, \frac{3}{8}$
- $\frac{5}{8}, \bar{y}, y + \frac{1}{4}$
 $y + \frac{1}{4}, \frac{5}{8}, \bar{y}$
 $\bar{y}, y + \frac{1}{4}, \frac{5}{8}$

no extra conditions

- 48 f 2 .. $x, 0, 0$
 $\frac{3}{4}, x + \frac{1}{4}, \frac{3}{4}$
- $\bar{x}, \frac{1}{2}, \frac{1}{2}$
 $\frac{1}{4}, \bar{x} + \frac{1}{4}, \frac{1}{4}$
- $0, x, 0$
 $x + \frac{3}{4}, \frac{1}{4}, \frac{3}{4}$
- $\frac{1}{2}, \bar{x}, \frac{1}{2}$
 $\bar{x} + \frac{3}{4}, \frac{3}{4}, \frac{1}{4}$
- $0, 0, x$
 $\frac{3}{4}, \frac{1}{4}, \bar{x} + \frac{3}{4}$
- $\frac{1}{2}, \frac{1}{2}, \bar{x}$
 $\frac{1}{4}, \frac{3}{4}, x + \frac{3}{4}$

 $hkl : h = 2n + 1$ or $h + k + l = 4n$

- 32 e . 3 . x, x, x
 $x + \frac{3}{4}, x + \frac{1}{4}, \bar{x} + \frac{3}{4}$
- $\bar{x}, \bar{x} + \frac{1}{2}, x + \frac{1}{2}$
 $\bar{x} + \frac{1}{4}, \bar{x} + \frac{1}{4}, \bar{x} + \frac{1}{4}$
- $\bar{x} + \frac{1}{2}, x + \frac{1}{2}, \bar{x}$
 $x + \frac{1}{4}, \bar{x} + \frac{3}{4}, x + \frac{3}{4}$
- $x + \frac{1}{2}, \bar{x}, \bar{x} + \frac{1}{2}$
 $\bar{x} + \frac{3}{4}, x + \frac{3}{4}, x + \frac{1}{4}$

 $0kl : k + l = 4n$

- 16 d . 3 2 $\frac{5}{8}, \frac{5}{8}, \frac{5}{8}$
 $\frac{1}{8}, \frac{1}{8}, \frac{1}{8}$
- $\frac{3}{8}, \frac{7}{8}, \frac{1}{8}$
 $\frac{7}{8}, \frac{3}{8}, \frac{5}{8}$
- $\frac{7}{8}, \frac{1}{8}, \frac{3}{8}$
 $\frac{5}{8}, \frac{7}{8}, \frac{3}{8}$

 $hkl : h = 2n + 1$
or $h, k, l = 4n + 2$
or $h, k, l = 4n$

- 8 b 2 3 . $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$
 $\frac{1}{4}, \frac{3}{4}, \frac{1}{4}$
- $0, 0, 0$
 $\frac{3}{4}, \frac{1}{4}, \frac{3}{4}$

 $hkl : h = 2n + 1$
or $h + k + l = 4n$

Symmetry of special projections

Along [001] $p4mm$ $\mathbf{a}' = \frac{1}{2}\mathbf{a}$ $\mathbf{b}' = \frac{1}{2}\mathbf{b}$
Origin at $\frac{1}{4}, 0, z$ Along [111] $p3m1$ $\mathbf{a}' = \frac{1}{6}(2\mathbf{a} - \mathbf{b} - \mathbf{c})$ $\mathbf{b}' = \frac{1}{6}(-\mathbf{a} + 2\mathbf{b} - \mathbf{c})$
Origin at x, x, x Along [110] $c2mm$ $\mathbf{a}' = \frac{1}{2}(-\mathbf{a} + \mathbf{b})$ $\mathbf{b}' = \mathbf{c}$
Origin at $x, x, \frac{1}{8}$

Maximal non-isomorphic subgroups

I	[2] $F_{23}1(F_{23}, 196)$	(1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12) +
	{ [3] $F_{4_1}12(I_{4_1}22, 98)$	(1; 2; 3; 4; 13; 14; 15; 16) +
	{ [3] $F_{4_1}12(I_{4_1}22, 98)$	(1; 2; 3; 4; 17; 18; 19; 20) +
	{ [3] $F_{4_1}12(I_{4_1}22, 98)$	(1; 2; 3; 4; 21; 22; 23; 24) +
	{ [4] $F_{13}2(R_{32}, 155)$	(1; 5; 9; 14; 19; 24) +
	{ [4] $F_{13}2(R_{32}, 155)$	(1; 6; 12; 13; 18; 24) +
	{ [4] $F_{13}2(R_{32}, 155)$	(1; 7; 10; 13; 19; 22) +
	{ [4] $F_{13}2(R_{32}, 155)$	(1; 8; 11; 14; 18; 22) +
IIa	{ [4] $P_{4_1}32(213)$	1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; 13; 14; 15; 16; 17; 18; 19; 20; 21; 22; 23; 24
	{ [4] $P_{4_1}32(213)$	1; 2; 3; 4; 13; 14; 15; 16; (9; 10; 11; 12; 17; 18; 19; 20) + (0, $\frac{1}{2}$, $\frac{1}{2}$) ; (5; 6; 7; 8; 21; 22; 23; 24) + ($\frac{1}{2}$, 0, $\frac{1}{2}$)
	{ [4] $P_{4_1}32(213)$	1; 2; 3; 4; 17; 18; 19; 20; (9; 10; 11; 12; 21; 22; 23; 24) + ($\frac{1}{2}$, 0, $\frac{1}{2}$) ; (5; 6; 7; 8; 13; 14; 15; 16) + ($\frac{1}{2}$, $\frac{1}{2}$, 0)
	{ [4] $P_{4_1}32(213)$	1; 2; 3; 4; 21; 22; 23; 24; (5; 6; 7; 8; 17; 18; 19; 20) + (0, $\frac{1}{2}$, $\frac{1}{2}$) ; (9; 10; 11; 12; 13; 14; 15; 16) + ($\frac{1}{2}$, $\frac{1}{2}$, 0)
	{ [4] $P_{4_3}32(212)$	1; 5; 9; 14; 19; 24; (4; 6; 11; 16; 18; 23) + (0, $\frac{1}{2}$, $\frac{1}{2}$) ; (3; 8; 10; 15; 20; 22) + ($\frac{1}{2}$, 0, $\frac{1}{2}$) ; (2; 7; 12; 13; 17; 21) + ($\frac{1}{2}$, $\frac{1}{2}$, 0)
	{ [4] $P_{4_3}32(212)$	1; 6; 12; 13; 18; 24; (4; 5; 10; 15; 19; 23) + (0, $\frac{1}{2}$, $\frac{1}{2}$) ; (3; 7; 11; 16; 17; 22) + ($\frac{1}{2}$, 0, $\frac{1}{2}$) ; (2; 8; 9; 14; 20; 21) + ($\frac{1}{2}$, $\frac{1}{2}$, 0)
	{ [4] $P_{4_3}32(212)$	1; 7; 10; 13; 19; 22; (4; 8; 12; 15; 18; 21) + (0, $\frac{1}{2}$, $\frac{1}{2}$) ; (3; 6; 9; 16; 20; 24) + ($\frac{1}{2}$, 0, $\frac{1}{2}$) ; (2; 5; 11; 14; 17; 23) + ($\frac{1}{2}$, $\frac{1}{2}$, 0)
	{ [4] $P_{4_3}32(212)$	1; 8; 11; 14; 18; 22; (4; 7; 9; 16; 19; 21) + (0, $\frac{1}{2}$, $\frac{1}{2}$) ; (3; 5; 12; 15; 17; 24) + ($\frac{1}{2}$, 0, $\frac{1}{2}$) ; (2; 6; 10; 13; 20; 23) + ($\frac{1}{2}$, $\frac{1}{2}$, 0)

IIb none**Maximal isomorphic subgroups of lowest index****IIc** [27] $F_{4_1}32(\mathbf{a}' = 3\mathbf{a}, \mathbf{b}' = 3\mathbf{b}, \mathbf{c}' = 3\mathbf{c})$ (210)**Minimal non-isomorphic supergroups****I** [2] $F d \bar{3}m$ (227); [2] $F d \bar{3}c$ (228)**II** [2] $P_{4_2}32(\mathbf{a}' = \frac{1}{2}\mathbf{a}, \mathbf{b}' = \frac{1}{2}\mathbf{b}, \mathbf{c}' = \frac{1}{2}\mathbf{c})$ (208)