

9. CRYSTAL LATTICES

Table 9.1.7.2. Three-dimensional Bravais lattices

Bravais lattice*	Lattice parameters		Metric tensor			Projections
	Conventional	Primitive	Conventional	Primitive/transf.†	Relations of the components	
<i>aP</i>	a, b, c α, β, γ	a, b, c α, β, γ	g_{11} g_{12} g_{13} g_{22} g_{23} g_{33}	g_{11} g_{12} g_{13} g_{22} g_{23} g_{33}		
<i>mP</i>	a, b, c $\beta, \alpha = \gamma = 90^\circ$	a, b, c $\beta, \alpha = \gamma = 90^\circ$	g_{11} 0 g_{13} g_{22} 0 g_{33}	g_{11} 0 g_{13} g_{22} 0 g_{33}		
<i>mC</i> (<i>mS</i>)	a, b, c $\beta, \alpha = \gamma = 90^\circ$	$a_1 = a_2, c$ $\gamma, \alpha = \beta$	g_{11} 0 g_{13} g_{22} 0 g_{33}	g'_{11} g'_{12} g'_{13} g'_{12} g'_{11} g'_{13} g'_{33}	P(C) $g'_{11} = \frac{1}{4}(g_{11} + g_{22})$ $g'_{12} = \frac{1}{4}(g_{11} - g_{22})$ $g'_{13} = \frac{1}{2}g_{13}$ $g_{11} = 2(g'_{11} + g'_{12})$ $g_{22} = 2(g'_{11} - g'_{12})$ $g_{13} = 2g'_{13}$	
<i>oP</i>	a, b, c $\alpha = \beta = \gamma = 90^\circ$	a, b, c $\alpha = \beta = \gamma = 90^\circ$	g_{11} 0 0 g_{22} 0 g_{33}	g_{11} 0 0 g_{22} 0 g_{33}		
<i>oC</i> (<i>oS</i>)	a, b, c $\alpha = \beta = \gamma = 90^\circ$	$a_1 = a_2, c$ $\gamma, \alpha = \beta = 90^\circ$	g_{11} 0 0 g_{22} 0 g_{33}	g'_{11} g'_{12} 0 g'_{12} g'_{11} 0 g'_{33}	P(C) $g'_{11} = \frac{1}{4}(g_{11} + g_{22})$ $g'_{12} = \frac{1}{4}(g_{11} - g_{22})$ $g_{11} = 2(g'_{11} + g'_{12})$ $g_{22} = 2(g'_{11} - g'_{12})$	
<i>oI</i>	a, b, c $\alpha = \beta = \gamma = 90^\circ$	$a_1 = a_2 = a_3$ α, β, γ $\cos \alpha + \cos \beta + \cos \gamma = -1$	g_{11} 0 0 g_{22} 0 g_{33}	$-\tilde{g}$ g'_{12} g'_{13} $-\tilde{g}$ g'_{23} $-\tilde{g}$ $\tilde{g} = g'_{12} + g'_{13} + g'_{23}$	P(I) $g'_{12} = \frac{1}{4}(-g_{11} - g_{22} + g_{33})$ $g'_{13} = \frac{1}{4}(-g_{11} + g_{22} - g_{33})$ $g'_{23} = \frac{1}{4}(g_{11} - g_{22} - g_{33})$ $g_{11} = -2(g'_{12} + g'_{13})$ $g_{22} = -2(g'_{12} + g'_{23})$ $g_{33} = -2(g'_{13} + g'_{23})$	
<i>oF</i>	a, b, c α, β, γ $\cos \alpha = \frac{-a^2 + b^2 + c^2}{2bc}$ $\cos \beta = \frac{a^2 + b^2 + c^2}{2ac}$ $\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab}$	a, b, c α, β, γ $\cos \alpha = \frac{-a^2 + b^2 + c^2}{2bc}$ $\cos \beta = \frac{a^2 + b^2 + c^2}{2ac}$ $\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab}$	g_{11} 0 0 g_{22} 0 g_{33}	\tilde{g}_1 g'_{12} g'_{13} \tilde{g}_2 g'_{23} \tilde{g}_3 $\tilde{g}_1 = g'_{12} + g'_{13}$ $\tilde{g}_2 = g'_{12} + g'_{23}$ $\tilde{g}_3 = g'_{13} + g'_{23}$	P(F) $g'_{12} = \frac{1}{4}g_{33}$ $g'_{13} = \frac{1}{4}g_{22}$ $g'_{23} = \frac{1}{4}g_{11}$ $g_{11} = 4g'_{23}$ $g_{22} = 4g'_{13}$ $g_{33} = 4g'_{12}$	

9.1. BASES, LATTICES AND BRAVAIS LATTICES

Table 9.1.7.2. Three-dimensional Bravais lattices (cont.)

Bravais lattice*	Lattice parameters		Metric tensor			Projections	
	Conventional	Primitive	Conventional	Primitive/transf. †	Relations of the components		
<i>tP</i>	$a_1 = a_2, c$ $\alpha = \beta = \gamma = 90^\circ$	$a_1 = a_2, c$ $\alpha = \beta = \gamma = 90^\circ$	g_{11} 0 0 g_{11} 0 g_{33}	g_{11} 0 0 g_{11} 0 g_{33}			
<i>tI</i>		$a_1 = a_2 = a_3$ $\gamma, \alpha = \beta$ $2 \cos \alpha + \cos \gamma = -1$	g_{11} 0 0 g_{11} 0 g_{33}	$P(I)$ \bar{g} g'_{12} g'_{13} \bar{g} g'_{13} \bar{g} $\bar{g} = -(g'_{12} + 2g'_{13})$	$g'_{12} = \frac{1}{4}(-2g_{11} + g_{33})$ $g'_{13} = -\frac{1}{4}g_{33}$ $g_{11} = 2(g'_{12} + g'_{13})$ $g_{33} = -4g'_{13}$		
<i>hR</i>	$a_1 = a_2, c$ $\alpha = \beta = 90^\circ$ $\gamma = 120^\circ$	$a_1 = a_2 = a_3$ $\alpha = \beta = \gamma$	g_{11} $-\frac{1}{2}g_{11}$ 0 g_{11} 0 g_{33}	$P(R)$ g'_{11} g'_{12} g'_{12} g'_{11} g'_{12} g'_{11}	$g'_{11} = \frac{1}{9}(3g_{11} + g_{33})$ $g'_{12} = \frac{1}{9}(-\frac{3}{2}g_{11} + g_{33})$ $g_{11} = 2(g'_{11} - g'_{12})$ $g_{33} = 3(g'_{11} + 2g'_{12})$		
<i>hP</i>		$a_1 = a_2, c$ $\alpha = \beta = 90^\circ$ $\gamma = 120^\circ$		g_{11} $-\frac{1}{2}g_{11}$ 0 g_{11} 0 g_{33}			
<i>cP</i>	$a_1 = a_2 = a_3$ $\alpha = \beta = \gamma = 90^\circ$	$a_1 = a_2 = a_3$ $\alpha = \beta = \gamma = 90^\circ$	g_{11} 0 0 g_{11} 0 g_{11}	g_{11} 0 0 g_{11} 0 g_{11}			
<i>cI</i>		$a_1 = a_2 = a_3$ $\alpha = \beta = \gamma = 109.5^\circ$ $\cos \alpha = -\frac{1}{3}$		$P(I)$ g'_{11} $-\frac{1}{3}g'_{11}$ $-\frac{1}{3}g'_{11}$ g'_{11} $-\frac{1}{3}g'_{11}$ g'_{11}	$g'_{11} = \frac{3}{4}g_{11}$ $g_{11} = \frac{4}{3}g'_{11}$		
<i>cF</i>		$a_1 = a_2 = a_3$ $\alpha = \beta = \gamma = 60^\circ$		$P(F)$ g'_{11} $\frac{1}{2}g'_{11}$ $\frac{1}{2}g'_{11}$ g'_{11} $\frac{1}{2}g'_{11}$ g'_{11}	$g'_{11} = \frac{1}{2}g_{11}$ $g_{11} = 2g'_{11}$		

* See footnote to Table 9.1.7.1. Symbols in parentheses are standard symbols, see Table 2.1.2.1.

† $P(C) = \frac{1}{2}(110/\bar{1}10/002)$, $P(I) = \frac{1}{2}(\bar{1}\bar{1}1/1\bar{1}1/11\bar{1})$, $P(F) = \frac{1}{2}(011/101/110)$, $P(R) = \frac{1}{3}(\bar{1}2\bar{1}/211/111)$.