result is

$$U' \simeq \begin{pmatrix} \frac{2KV}{(R-L)} & 0 & 0 & \frac{-K(R+L)}{(R-L)} \\ 0 & \frac{2KV}{(T-B)} & 0 & \frac{-K(T+B)}{(T-B)} \\ 0 & 0 & \frac{KV}{(F-N)} & \frac{-KN}{(F-N)} \\ 0 & 0 & 0 & K \end{pmatrix},$$

which is the orthographic window.

It may be convenient in some applications to separate the functions of windowing and the application of perspective, and to write

$$U = U'P,$$

where U and U' are as above and P is a perspective transformation given by

$$P = (U')^{-1}U \simeq \begin{pmatrix} S-E & 0 & C & -SC/V \\ 0 & S-E & D & -SD/V \\ 0 & 0 & F-E+N & -NF/V \\ 0 & 0 & V & -E \end{pmatrix},$$

which involves F and N but not R, L, T or B. In this form the action of P may be thought of as compressing distant parts of display space prior to an orthographic projection by U' into picture space.

Other factorizations of U are possible, for example

$$U = U''P'$$

with

$$U'' \simeq \begin{pmatrix} \frac{2KV}{R-L} & 0 & 0 & \frac{-K(R+L)}{(R-L)} \\ 0 & \frac{2KV}{T-B} & 0 & \frac{-K(T+B)}{(T-B)} \\ 0 & 0 & \frac{KV(N-E)(F-E)}{E^2(F-N)} & \frac{KN(F-E)}{E(F-N)} \\ 0 & 0 & 0 & K \end{pmatrix}$$

$$P' \simeq \begin{pmatrix} S-E & 0 & C & -SC/V \\ 0 & S-E & D & -SD/V \\ 0 & 0 & -E & 0 \\ 0 & 0 & V & -E \end{pmatrix},$$

which renders P' independent of all six boundary planes, but U'' is no longer independent of E. It is not possible to factorize U so that the left factor is a function only of the boundary planes and the right factor a function only of eye and screen positions.

Note that as $E \to -\infty$, $U'' \to U'$, P and $P' \to -IE \simeq I$.

3.3.1.3.6. *Stereoviews*

Assuming that left- and right-eye views are to be presented through the same viewport (next section) or that their viewports are to be superimposed by an external optical system, *e.g.* Ortony mirrors, then stereopairs are obtained by using appropriate eye coordinates in the U matrix of the previous section. However, U may be factorized according to

$$U = U'''S$$

in which U''' is the matrix U obtained by setting (C, D, E, V) to correspond to the point midway between the viewer's eyes and

$$S = \begin{pmatrix} 1 & 0 & c/(S-E) & -cS/(S-E)V \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\simeq \begin{pmatrix} V & 0 & cV/(S-E) & -cS/(S-E) \\ 0 & V & 0 & 0 \\ 0 & 0 & V & 0 \\ 0 & 0 & 0 & V \end{pmatrix}$$

in which $(c, 0, 0, V)$ is the position of the right eye relative to the mean eye position, and the left-eye view is obtained by negating c.

Stereo is often approximated by introducing a rotation about the Y axis of $\pm\sin^{-1}[c/(S-E)]$ to the views or $\sin^{-1}[2c/(S-E)]$ to one of them. The first corresponds to

$$S = \begin{pmatrix} \sqrt{1-\sigma^2} & 0 & \sigma & 0 \\ 0 & 1 & 0 & 0 \\ -\sigma & 0 & \sqrt{1-\sigma^2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

with $\sigma = c/(S-E)$. The main difference is in the resulting Z value, which only affects depth cueing and z clipping. The X translation which arises if $S \neq 0$ is also suppressed, but this is not likely to be noticeable. σ is often treated as a constant, such as $\sin 3°$.

The distinction in principle between the true S and the rotational approximation is that with the true S the eye moves relative to the screen and the displayed object, whereas with the approximation the eye and the screen are moved relative to the displayed object, in going from one view to the other.

Strobing of left and right images may conveniently be accomplished with an electro-optic liquid-crystal shutter as described by Harris *et al.* (1985). The shutter is switched by the display itself, thus solving the synchronization problem in a manner free of inertia.

A further discussion of stereopairs may be found in Johnson (1970) and in Thomas (1993), the second of which generalizes the treatment to allow for the possible presence of an optical system.

3.3.1.3.7. *Viewports*

The window transformation of the previous two sections has been constructed to yield picture coordinates (X, Y, Z, W) (formerly called x, y, z, w) such that a point having X/W or $Y/W = \pm 1$ is on the boundary of the picture, and the clipping hardware operates on this basis. However, the edges of the picture need not be at the edges of the screen and a viewport transformation, V, is therefore needed to position the picture in the requisite part of the screen.

$$V = \begin{pmatrix} (r-l)/2 & 0 & 0 & (r+l)/2 \\ 0 & (t-b)/2 & 0 & (t+b)/2 \\ 0 & 0 & n & 0 \\ 0 & 0 & 0 & n \end{pmatrix},$$

where r, l, t and b are now the right, left, top and bottom boundaries of the picture area, or viewport, expressed in screen coordinates, and n is the full-screen deflection value. Thus a point with $X/W = 1$ in picture space plots on the screen with an X coordinate which is a fraction r/n of full-screen deflection to the right. Z/W is unchanged

references