

1. CRYSTAL GEOMETRY AND SYMMETRY

1.4.1.2. Arithmetic crystal classes in one, two and higher dimensions

In one dimension, there are two geometric crystal classes, 1 and  $m$ , and a single Bravais lattice,  $\bar{1}$ . Two arithmetic crystal classes result,  $\bar{1}$  and  $m\bar{1}$ . In two dimensions, there are ten geometric crystal classes, and two Bravais lattices,  $p$  and  $c$ ; 13 arithmetic

crystal classes result. The two-dimensional geometric and arithmetic crystal classes are listed in Table 1.4.1.1.

The number of arithmetic crystal classes increases rapidly with increasing dimensionality; there are 710 (plus 70 enantiomorphs) in four dimensions (Brown, Bülow, Neubüser, Wondratschek & Zassenhaus, 1978), but those in dimensions higher than three are not needed in this volume.

Table 1.4.2.1. The three-dimensional space groups, arranged by arithmetic crystal class; in a few geometric crystal classes this differs somewhat from the conventional numerical order; see International Tables Volume A, p. 728

Crystal system	Crystal class			Space group	
	Geometric	Arithmetic		Number	Symbol
		Number	Symbol		
Triclinic	$\bar{1}$	1	$\bar{1}P$	1	$P\bar{1}$
		2	$\bar{1}P$	2	$P\bar{1}$
Monoclinic	2	3	$2P$	3	$P2$
		4	$2C$	4	$P2_1$
		5	$mP$	5	$C2$
		6	$mC$	6	$Pm$
	$m$	7		7	$Pc$
		8		8	$Cm$
		9		9	$Cc$
		10	$2/mP$	10	$P2/m$
$2/m$	11		11	$P2_1/m$	
	13		13	$P2/c$	
	14		14	$P2_1/c$	
	12	$2/mC$	12	$C2/m$	
15		15	$C2/c$		
Orthorhombic	222	9	$222P$	16	$P222$
				17	$P222_1$
				18	$P2_12_12$
		10	$222C$	19	$P2_12_12_1$
				20	$C222_1$
				21	$C222$
				22	$F222$
		$mm$	$222I$	23	$I222$
				24	$I2_12_12_1$
	25			$Pmm2$	
	26			$Pmc2_1$	
	13	$mm2P$	27	$Pcc2$	
			28	$Pma2$	
			29	$Pca2_1$	
			30	$Pnc2$	
			31	$Pmn2_1$	
			32	$Pba2$	
33			$Pna2_1$		
34			$Pnn2$		
14	$mm2C$	35	$Cmm2$		
		36	$Cmc2_1$		
		37	$Ccc2$		
15	$2mmC$ ( $Amm2$ )	38	$C2mm$ ( $Amm2$ )		
		39	$C2me$ ( $Aem2$ )		
		40	$C2cm$ ( $Ama2$ )		
16	$mm2F$	41	$C2ce$ ( $Aea2$ )		
		42	$Fmm2$		
		43	$Fdd2$		
17	$mm2I$	44	$Imm2$		
		45	$Iba2$		
		46	$Ima2$		

1.4. ARITHMETIC CRYSTAL CLASSES AND SYMMORPHIC SPACE GROUPS

Table 1.4.2.1. Three-dimensional space groups (cont.)

Crystal system	Crystal class			Space group				
	Geometric	Arithmetic		Number	Symbol			
		Number	Symbol					
Orthorhombic (cont.)	<i>mmm</i>	18	<i>mmmP</i>	47	<i>Pmmm</i>			
				48	<i>Pnnn</i>			
				49	<i>Pccm</i>			
				50	<i>Pban</i>			
				51	<i>Pmma</i>			
				52	<i>Pnna</i>			
				53	<i>Pmna</i>			
				54	<i>Pcca</i>			
				55	<i>Pbam</i>			
				56	<i>Pccn</i>			
				57	<i>Pbcm</i>			
				58	<i>Pnnm</i>			
				59	<i>Pmnn</i>			
				60	<i>Pbcn</i>			
				61	<i>Pbca</i>			
				62	<i>Pnma</i>			
				19	<i>mmmC</i>	63	<i>Cmcm</i>	
						64	<i>Cmce</i>	
	65	<i>Cmmm</i>						
	20	<i>mmmF</i>	66	<i>Cccm</i>				
			67	<i>Cmme</i>				
21	<i>mmmI</i>	68	<i>Ccce</i>					
		69	<i>Fmmm</i>					
		70	<i>Fddd</i>					
		71	<i>Immm</i>					
Tetragonal	4	22	4 <i>P</i>	72	<i>Ibca</i>			
				73	<i>Imma</i>			
				74				
				75	<i>P4</i>			
				76	<i>P4<sub>1</sub></i>			
				77	<i>P4<sub>2</sub></i>			
				78	<i>P4<sub>3</sub></i>			
				79	<i>I4</i>			
				80	<i>I4<sub>1</sub></i>			
				4̄	23	4̄ <i>I</i>	81	<i>P4</i>
							82	<i>I4</i>
	83	<i>P4/m</i>						
	84	<i>P4<sub>2</sub>/m</i>						
	85	<i>P4/n</i>						
	86	<i>P4<sub>2</sub>/n</i>						
	87	<i>I4/m</i>						
	88	<i>I4<sub>1</sub>/a</i>						
	422	24	422 <i>P</i>				89	<i>P422</i>
							90	<i>P42<sub>1</sub>2</i>
	4/m	25	4/m <i>P</i>	91	<i>P4<sub>1</sub>22</i>			
				92	<i>P4<sub>1</sub>2<sub>1</sub>2</i>			
93				<i>P4<sub>2</sub>22</i>				
94				<i>P4<sub>2</sub>2<sub>1</sub>2</i>				
95				<i>P4<sub>3</sub>22</i>				
96				<i>P4<sub>3</sub>2<sub>1</sub>2</i>				
97				<i>I422</i>				
98				<i>I4<sub>1</sub>22</i>				
4mm				26	4mm <i>P</i>	99	<i>P4mm</i>	
						100	<i>P4bm</i>	
	101	<i>P4<sub>2</sub>cm</i>						
	102	<i>P4<sub>2</sub>nm</i>						
	103	<i>P4cc</i>						
	104	<i>P4nc</i>						
	105	<i>P4<sub>2</sub>mc</i>						
	106	<i>P4<sub>2</sub>bc</i>						
	31	27	4mm <i>I</i>			107	<i>I4mm</i>	
						108	<i>I4cm</i>	
109				<i>I4<sub>1</sub>md</i>				
110				<i>I4<sub>1</sub>cd</i>				

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Table 1.4.2.1. *Three-dimensional space groups (cont.)*

Crystal system	Crystal class			Space group		
	Geometric	Arithmetic				
		Number	Symbol	Number	Symbol	
Tetragonal ( <i>cont.</i> )	$\bar{4}m$	32	$\bar{4}2mP$	111	$P\bar{4}2m$	
				112	$P\bar{4}2c$	
		33	$\bar{4}m2P$	113	$P\bar{4}2_1m$	
				114	$P\bar{4}2_1c$	
				115	$P\bar{4}m2$	
				116	$P\bar{4}c2$	
				117	$P\bar{4}b2$	
				118	$P\bar{4}n2$	
		34	$\bar{4}m2I$	119	$I\bar{4}m2$	
				120	$I\bar{4}c2$	
	4/ <i>mmm</i>	36	4/ <i>mmmP</i>	121	$I\bar{4}2m$	
				122	$I\bar{4}2d$	
				123	$P4/mmm$	
				124	$P4/mcc$	
				125	$P4/nbm$	
				126	$P4/nnc$	
				127	$P4/mbm$	
	37	4/ <i>mmmI</i>		128	$P4/mnc$	
				129	$P4/nmm$	
				130	$P4/ncc$	
131				$P4_2/mmc$		
132				$P4_2/mcm$		
133				$P4_2/nbc$		
134				$P4_2/nnm$		
135				$P4_2/mbc$		
136				$P4_2/mmm$		
137				$P4_2/nmc$		
Trigonal	3	38	3 <i>P</i>	143	<i>P</i> 3	
				144	$P3_1$	
				145	$P3_2$	
				146	<i>R</i> 3	
				147	$\bar{3}$	
	$\bar{3}$	40	$\bar{3}P$		148	$R\bar{3}$
					149	$P312$
					151	$P3_112$
					153	$P3_212$
					150	$P321$
	32	42	312 <i>P</i>		152	$P3_121$
					154	$P3_221$
					155	<i>R</i> 32
					156	$P3m1$
					158	$P3c1$
	3 <i>m</i>	45	3 <i>mP</i>		157	$P31m$
					159	$P31c$
					160	<i>R</i> 3 <i>m</i>
					161	$R\bar{3}c$
					162	$P\bar{3}1m$
$\bar{3}m$	48	$\bar{3}1mP$		163	$P\bar{3}1c$	
				164	$P\bar{3}m1$	
				165	$P\bar{3}c1$	
				166	$R\bar{3}m$	
				167	$R\bar{3}c$	

