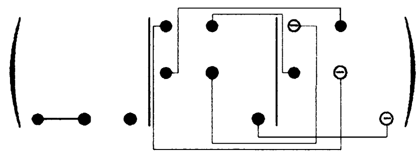
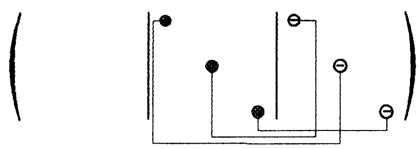
International Tables for Crystallography (2006). Vol. D, Section 1.1.4.8.7, pp. 19–20.



There are 7 independent components.

1.1.4.8.5.2. *Group* 422

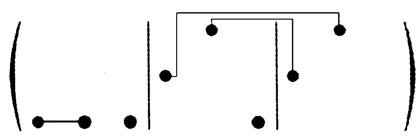
One combines the reductions for groups 4 and 222:



There are 3 independent components.

1.1.4.8.5.3. *Group* 4*mm*

One combines the reductions for groups 4 and 2*m*:



There are 4 independent components.

1.1.4.8.5.4. *Group* 4/*m*

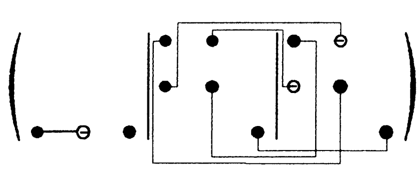
All the components are equal to zero.

1.1.4.8.5.5. *Group* $\bar{4}$

The matrix corresponding to axis $\bar{4}$ is

$$\begin{pmatrix} 0 & \bar{1} & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \bar{1} \end{pmatrix}$$

and the form of the 3×9 matrix is

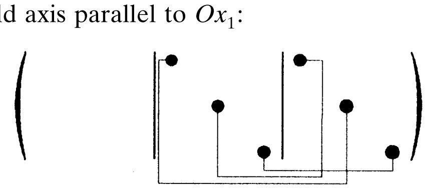


There are 6 independent components.

1.1.4.8.5.6. *Group* $\bar{4}2m$

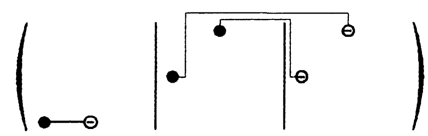
One combines either the reductions for groups $\bar{4}$ and 222, or the reductions for groups $\bar{4}$ and 2*mm*.

(i) Twofold axis parallel to Ox_1:



There are 6 independent components.

(ii) Mirror perpendicular to Ox_1 (the twofold axis is at 45°)



The number of independent components is of course the same, 6.

1.1.4.8.5.7. *Group* 4/*mm*

All the components are equal to zero.

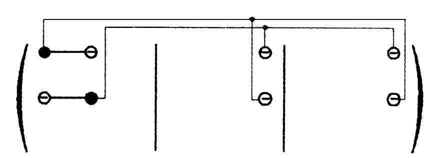
1.1.4.8.6. *Hexagonal and cylindrical systems*

1.1.4.8.6.1. *Groups* 6, A_∞, 622, $A_\infty \infty A_2$, 6*mm and* $A_\infty \infty M$

It was shown in Section 1.1.4.6.2.3 that, in the case of tensors of rank 3, the reduction is the same for axes of order 4, 6 or higher. The reduction will then be the same as for the tetragonal system.

1.1.4.8.6.2. *Group* $\bar{6} = 3/m$

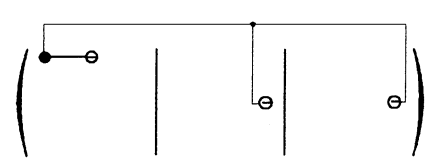
One combines the reductions for the groups corresponding to a threefold axis parallel to Ox_3 and to a mirror perpendicular to Ox_3:



There are 2 independent components.

1.1.4.8.6.3. *Group* $\bar{6}2m$

One combines the reductions for groups 6 and 2*mm*:



There is 1 independent component.

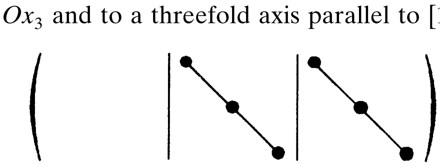
1.1.4.8.6.4. *Groups* 6/*m*, $(A_\infty/M)C$, 6/*mm and* $(A_\infty/M)\infty(A_2/M)C$

All the components are equal to zero.

1.1.4.8.7. *Cubic and spherical systems*

1.1.4.8.7.1. *Group* 23

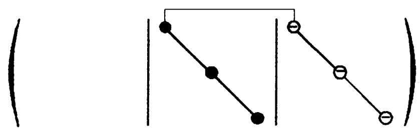
One combines the reductions corresponding to a twofold axis parallel to Ox_3 and to a threefold axis parallel to [111]:



There are 2 independent components.

1.1.4.8.7.2. *Groups* 432 *and* $\infty A_\infty/M$

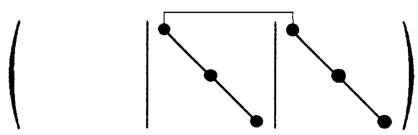
One combines the reductions corresponding to groups 422 and 23:



There is 1 independent component.

1.1.4.8.7.3. *Group* $\bar{4}3m$

One combines the reductions corresponding to groups $\bar{4}2m$ and 23:



There is 1 independent component.

1.1.4.8.7.4. *Groups* $m\bar{3}$, $m\bar{3}m$ *and* $\infty(A_\infty/M)C$

All the components are equal to zero.

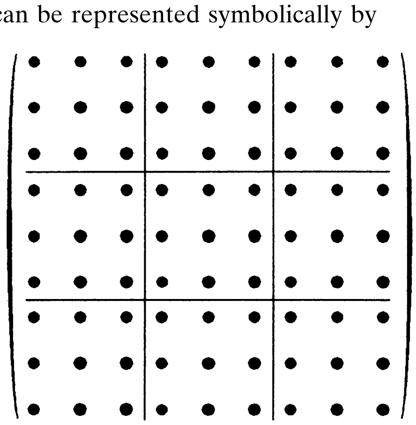
1.1.4.9. *Reduction of the components of a tensor of rank 4*

1.1.4.9.1. *Triclinic system (groups* $\bar{1}$, 1*)*

There is no reduction; all the components are independent. Their number is equal to 81. They are usually represented as a 9×9 matrix, where components t_{ijkl} are replaced by *ijkl*, for brevity:

kl / *ij*	11	22	33	23	31	12	32	13	21
11	1111	1122	1133	1123	1131	1112	1132	1113	1121
22	2211	2222	2233	2223	2231	2212	2232	2213	2221
33	3311	3322	3333	3323	3331	3312	3332	3313	3321
23	2311	2322	2333	2323	2331	2312	2332	2313	2321
31	3111	3122	3133	3123	3131	3112	3132	3113	3121
12	1211	1222	1233	1223	1231	1212	1232	1213	1221
32	3211	3222	3233	3223	3231	3212	3232	3213	3221
13	1311	1322	1333	1323	1331	1312	1332	1313	1321
21	2111	2122	2133	2123	2131	2112	2132	2113	2121

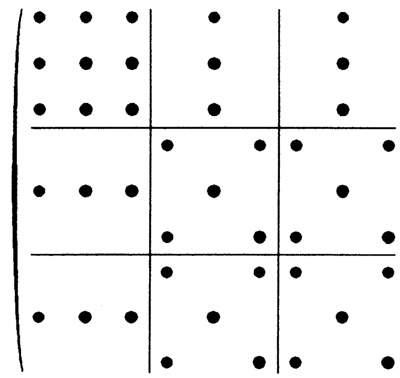
This matrix can be represented symbolically by



where the 9×9 matrix has been subdivided for clarity in to nine 3×3 submatrices.

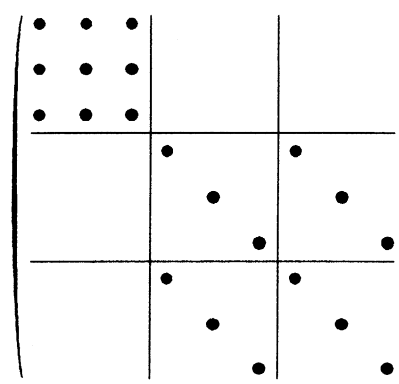
1.1.4.9.2. *Monoclinic system (groups* $2/m$, 2, *m)*

The reduction is obtained by the method of direct inspection. For a twofold axis parallel to Ox_2, one finds



There are 41 independent components.

1.1.4.9.3. *Orthorhombic system (groups mmm,* 2*mm,* 222*)*



There are 21 independent components.

1.1.4.9.4. *Trigonal system*

1.1.4.9.4.1. *Groups* 3 *and* $\bar{3}$

The reduction is first applied in the system of axes tied to the eigenvectors of the operator representing a threefold axis. The system of axes is then changed to a system of orthonormal axes with Ox_3 parallel to the threefold axis:

kl / *ij*	11	22	33	23	31	12	32	13	21
11	1111	1122	1133	1123	−2231	1112	1132	−2213	1121
22	1122	1111	1133	−1123	2231	−1121	−1132	2213	−1112
33	3311	3311	3333			3312			−3312
23	2311	−2311		2323	2331	1322	2332	2313	1322
31	−3122	3122		3123	3131	3211	3132	3113	3211
12	1211	−2111	1233	2213	1132	1212	2231	1123	1221
32	3211	−3211		3113	−3132	3122	3131	−3123	3122
13	−1322	1322		−2313	2332	2311	−2331	2323	2311
21	2111	−121	−1233	2213	1132	1221	2231	1123	1212

with

$$\left.\begin{aligned} t_{1111} - t_{1122} &= t_{1212} + t_{1221} \\ t_{1112} + t_{1121} &= -(t_{1211} + t_{2111}). \end{aligned}\right\}$$

There are 27 independent components.

references