

1.1. INTRODUCTION TO THE PROPERTIES OF TENSORS

1.1.4.9.4.2. Groups $\bar{3}m$, 32 , $3m$, with the twofold axis parallel to Ox_1

kl	11	22	33	23	31	12	32	13	21
ij									
11	1111	1122	1133	1123			1132		
22	1122	1111	1133	-1123			-1132		
33	3311	3311	3333						
23	2311	-2311		2323			2332		
31					3131	3211		3113	3211
12					1132	1212		1123	1221
32	3211	-3211		3113			3131		
13					2332	2311		2323	2311
21					1132	1221		1123	1212

with

$$t_{1111} - t_{1122} = t_{1212} + t_{1221}.$$

There are 14 independent components.

1.1.4.9.5. Tetragonal system

1.1.4.9.5.1. Groups $4/m$, $4, \bar{4}$

kl	11	22	33	23	31	12	32	13	21
ij									
11	1111	1122	1133			1112			-2212
22	1122	1111	1133			2212			-1112
33	3311	3311	3333			3312			-3312
23				2323	2331		2332	2313	
31				3123	3131		3132	3113	
12	1211	1222	1233			1212			1221
32				3113	-3132		3131	-3123	
13				-2313	2332		-2331	2323	
21	-1222	-1211	-1233						1212

There are 21 independent components.

1.1.4.9.5.2. Groups $4/m\bar{m}2$, 422 , $4mm$, $\bar{4}2m$

kl	11	22	33	23	31	12	32	13	21
ij									
11	1111	1122	1133						
22	1122	1111	1133						
33	3311	3311	3333						
23				2323			2332		
31					3131			3113	
12						1212			1221
32				3113			3131		
13					2332			2323	
21						1221			1212

There are 11 independent components.

1.1.4.9.6. Hexagonal and cylindrical systems

1.1.4.9.6.1. Groups $6/m$, $\bar{6}$, 6 ; $(A_\infty/M)C$, A_∞

kl	11	22	33	23	31	12	32	13	21
ij									
11	1111	1122	1133			1112			1121
22	1122	1111	1133			-1121			-1112
33	3311	3311	3333			3312			-3312
23				2323	2331		2332	2313	
31				3123	3131		3132	3113	
12	1211	-2111	1233			1212			1221
32				3113	-3132		3131	-3123	
13				-2313	2332		-2331	2323	
21	2111	-1211	-1233			1132	1221		1123

with

$$\left. \begin{aligned} t_{1111} - t_{1122} &= t_{1212} + t_{1221} \\ t_{1112} + t_{1121} &= -(t_{1211} + t_{2111}). \end{aligned} \right\}$$

There are 19 independent components.

1.1.4.9.6.2. Groups $6/m\bar{m}2$, 622 , $6mm$, $\bar{6}2m$; $(A_\infty/M)\infty$; $(A_2/M)C$, $A_\infty\infty A_2$

kl	11	22	33	23	31	12	32	13	21
ij									
11	1111	1122	1133						
22	1122	1111	1133						
33	3311	3311	3333						
23				2323			2332		
31					3131			3113	
12						1212			1221
32				3113			3131		
13					2332			2323	
21						1221			1212

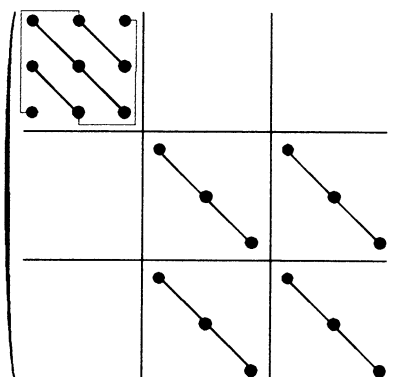
with

$$t_{1111} - t_{1122} = t_{1212} + t_{1221}.$$

There are 11 independent components.

1.1.4.9.7. Cubic system

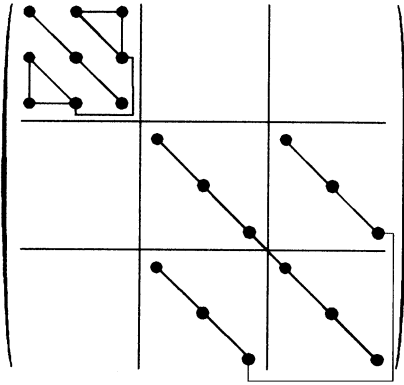
1.1.4.9.7.1. Groups 23 , $\bar{3}m$



There are 7 independent components.

1. TENSORIAL ASPECTS OF PHYSICAL PROPERTIES

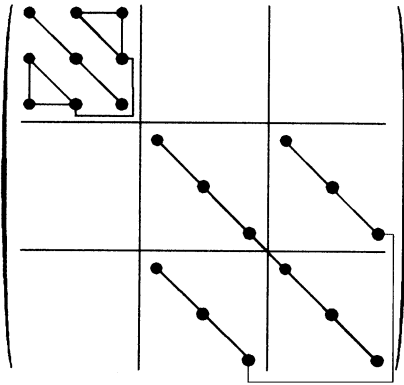
1.1.4.9.7.2. Groups $m\bar{3}m$, 432 , $\bar{4}3m$



There are 4 independent components. The tensor is symmetric.

1.1.4.9.8. Spherical system

1.1.4.9.8.1. Groups $\infty(A_\infty/M)C$ and ∞A_∞



with

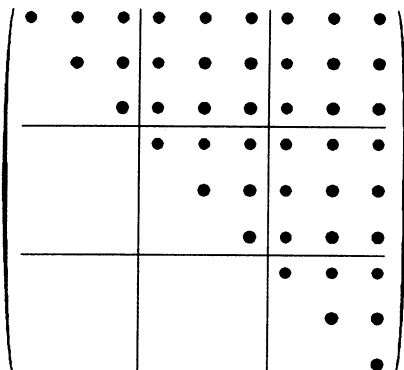
$$t_{1111} - t_{1122} = t_{1212} + t_{1221}.$$

There are 3 independent components. The tensor is symmetric.

1.1.4.9.9. Symmetric tensors of rank 4

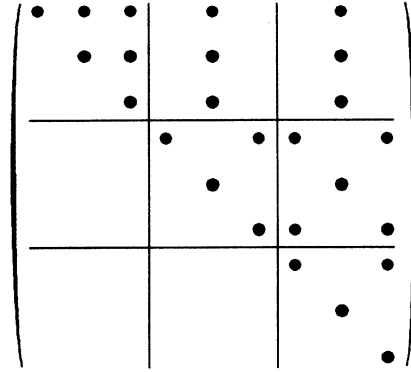
For symmetric tensors such as those representing principal properties, one finds the following, representing the nonzero components for the leading diagonal and for one half of the others.

1.1.4.9.9.1. Triclinic system



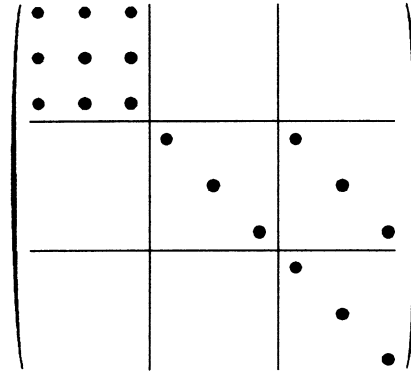
There are 45 independent coefficients.

1.1.4.9.9.2. Monoclinic system



There are 25 independent coefficients.

1.1.4.9.9.3. Orthorhombic system



There are 15 independent coefficients.

1.1.4.9.9.4. Trigonal system

(i) Groups 3 and $\bar{3}$

kl	11	22	33	23	31	12	32	13	21
ij									
11	1111	1122	1133	1123	-2231	1112	1132	-2213	-1112
22		1111	1133	-1123	2231	-1121	-1132	2213	-1112
33			3333			3312			-3312
23				2323	2331	2213	2332		2213
31					3131	1132		2332	1132
12						1212	2231	1123	1221
32							3131	-2331	2231
13								2323	1123
21									1212

with

$$t_{1111} - t_{1122} = t_{1212} + t_{1221}.$$

There are 15 independent components.