Table 1.7.3.3. *Classes of refractive-index inequalities for collinear phase matching of three-wave interactions in positive and negative uniaxial crystals*

Types I, II and III refer to SFG; the types of the corresponding DFG are given in Table 1.7.3.1 (Fève *et al.*, 1993).

Positive sign ($n_e > n_o$)	Negative sign ($n_o > n_e$)	Types of SFG
$\frac{n_{o3}}{\lambda_3} < \frac{n_{o1}}{\lambda_1} + \frac{n_{e2}}{\lambda_2}; \frac{n_{e1}}{\lambda_1} + \frac{n_{o2}}{\lambda_2}$	$\frac{n_{o1}}{\lambda_1} + \frac{n_{e2}}{\lambda_2}, \frac{n_{e1}}{\lambda_1} + \frac{n_{o2}}{\lambda_2} < \frac{n_{e3}}{\lambda_3}$	I, II, III
$\frac{n_{e1}}{\lambda_1} + \frac{n_{o2}}{\lambda_2} < \frac{n_{o3}}{\lambda_3} < \frac{n_{o1}}{\lambda_1} + \frac{n_{e2}}{\lambda_2}$	$\frac{n_{o1}}{\lambda_1} + \frac{n_{e2}}{\lambda_2} < \frac{n_{e3}}{\lambda_3} < \frac{n_{e1}}{\lambda_1} + \frac{n_{o2}}{\lambda_2}$	I, II
$\frac{n_{o1}}{\lambda_1} + \frac{n_{e2}}{\lambda_2} < \frac{n_{o3}}{\lambda_3} < \frac{n_{e1}}{\lambda_1} + \frac{n_{o2}}{\lambda_2}$	$\frac{n_{e1}}{\lambda_1} + \frac{n_{o2}}{\lambda_2} < \frac{n_{e3}}{\lambda_3} < \frac{n_{o1}}{\lambda_1} + \frac{n_{e2}}{\lambda_2}$	I, III
$\frac{n_{o1}}{\lambda_1} + \frac{n_{e2}}{\lambda_2}, \frac{n_{e1}}{\lambda_1} + \frac{n_{o2}}{\lambda_2} < \frac{n_{o3}}{\lambda_3} < \frac{n_{e_1}}{\lambda_1} + \frac{n_{e2}}{\lambda_2}$	$\frac{n_{o1}}{\lambda_1} + \frac{n_{e2}}{\lambda_2}, \frac{n_{e1}}{\lambda_1} + \frac{n_{o2}}{\lambda_2} < \frac{n_{e3}}{\lambda_3} < \frac{n_{o_1}}{\lambda_1} + \frac{n_{o2}}{\lambda_2}$	I
$\frac{n_{e_1}}{\lambda_1} + \frac{n_{e2}}{\lambda_2} < \frac{n_{o3}}{\lambda_3}$	$\frac{n_{o_1}}{\lambda_1} + \frac{n_{o2}}{\lambda_2} < \frac{n_{e3}}{\lambda_3}$	None

Table 1.7.3.4. *Classes of refractive-index inequalities for collinear phase matching of four-wave interactions in positive ($n_a = n_e, n_b = n_o$) and negative ($n_a = n_o, n_b = n_e$) uniaxial crystals with $(n_{b4}/\lambda_4) < (n_{a1}/\lambda_1) + (n_{a2}/\lambda_2) + (n_{a3}/\lambda_3)$*

If this inequality is not verified, no phase matching is allowed. The types of phase matching refer to SFG; the types of the corresponding DFG are given in Table 1.7.3.2 (Fève, 1994).

Positive sign ($n_e > n_o$)	Negative sign ($n_o > n_e$)	Types of SFG
$\frac{n_{a1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}, \frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}, \frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4}$		I
$\frac{n_{a1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}, \frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4} < \frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}$		I, V^4
$\frac{n_{a1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}, \frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4} < \frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}$		I, VI^4
$\frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}, \frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4} < \frac{n_{a1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}$		I, VII^4
$\frac{n_{a1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4} < \frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}, \frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}$	$\frac{n_{b1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4}$	I, V^4, VI^4
	$\frac{n_{b4}}{\lambda_4} < \frac{n_{b1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}$	I, II, V^4, VI^4
$\frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4} < \frac{n_{a1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}, \frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}$	$\frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4}$	I, V^4, VII^4
	$\frac{n_{b4}}{\lambda_4} < \frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}$	I, III, V^4, VII^4
$\frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4} < \frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}, \frac{n_{a1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}$	$\frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4}$	I, VI^4, VII^4
	$\frac{n_{b4}}{\lambda_4} < \frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}$	I, IV, VI^4, VII^4
$\frac{n_{b4}}{\lambda_4} < \frac{n_{a1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}, \frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}, \frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}$	$\frac{n_{b1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}, \frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}, \frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4}$	I, V^4, VI^4, VII^4
	$\frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}, \frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4} < \frac{n_{b1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}$	I, II, V^4, VI^4, VII^4
	$\frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}, \frac{n_{b1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4} < \frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}$	I, III, V^4, VI^4, VII^4
	$\frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}, \frac{n_{b1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4} < \frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}$	I, IV, V^4, VI^4, VII^4
	$\frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4} < \frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}, \frac{n_{b1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}$	I, II, III, V^4, VI^4, VII^4
	$\frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4} < \frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}, \frac{n_{b1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}$	I, II, IV, V^4, VI^4, VII^4
	$\frac{n_{b1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3} < \frac{n_{b4}}{\lambda_4} < \frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}, \frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}$	I, III, IV, V^4, VI^4, VII^4
	$\frac{n_{b4}}{\lambda_4} < \frac{n_{a1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}, \frac{n_{b1}}{\lambda_1} + \frac{n_{a2}}{\lambda_2} + \frac{n_{b3}}{\lambda_3}, \frac{n_{b1}}{\lambda_1} + \frac{n_{b2}}{\lambda_2} + \frac{n_{a3}}{\lambda_3}$	All